For each subspace in Exercises 1–8, (a) find a basis for the subspace, and (b) state the dimension.

1. $\left\{\begin{bmatrix} s-2t \\ s+t \\ 3t \end{bmatrix} : s, t \text{ in } \mathbb{R}\right\}$ **2.** $\left\{\begin{bmatrix} 2a \\ -4b \\ -2a \end{bmatrix} : a, b \text{ in } \mathbb{R}\right\}$

3. $\left\{\begin{bmatrix} 2c \\ a-b \\ b-3c \\ a+2b \end{bmatrix} : a, b, c \text{ in } \mathbb{R}\right\}$ **4.** $\left\{\begin{bmatrix} p+2q \\ -p \\ 3p-q \\ p+q \end{bmatrix} : p, q \text{ in } \mathbb{R}\right\}$

In Exercises 11 and 12, find the dimension of the subspace spanned by the given vectors.

11. $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix}$

12. $\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -6 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ 5 \end{bmatrix}$

Determine the dimensions of Nul A and Col A for the matrices shown in Exercises 13–18.

13. $A = \begin{bmatrix} 1 & -6 & 9 & 0 & -2 \\ 0 & 1 & 2 & -4 & 5 \\ 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

21. The first four Hermite polynomials are $1, 2t, -2 + 4t^2$, and $-12t + 8t^3$. These polynomials arise naturally in the study of certain important differential equations in mathematical physics.[2] Show that the first four Hermite polynomials form a basis of $\mathbb{P}_3$.

23. Let $\mathcal{B}$ be the basis of $\mathbb{P}_3$ consisting of the Hermite polynomials in Exercise 21, and let $\mathbf{p}(t) = -1 + 8t^2 + 8t^3$. Find the coordinate vector of $\mathbf{p}$ relative to $\mathcal{B}$.

27. Explain why the space $\mathbb{P}$ of all polynomials is an infinite-dimensional space.

In Exercises 29 and 30, V is a nonzero finite-dimensional vector space, and the vectors listed belong to V. Mark each statement True or False. Justify each answer. (These questions are more difficult than those in Exercises 19 and 20.)

29. a. If there exists a set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ that spans V, then $\dim V \leq p$.

b. If there exists a linearly independent set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ in V, then $\dim V \geq p$.

c. If $\dim V = p$, then there exists a spanning set of $p + 1$ vectors in V.

In Exercises 1–4, assume that the matrix A is row equivalent to B. Without calculations, list rank A and dim Nul A. Then find bases for Col A, Row A, and Nul A.

1. $A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$,

$B = \begin{bmatrix} 1 & 0 & -1 & 5 \\ 0 & -2 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 3 & 4 & -1 & 2 \\ 2 & 6 & 6 & 0 & -3 \\ 3 & 9 & 3 & 6 & -3 \\ 3 & 9 & 0 & 9 & 0 \end{bmatrix}$,

$B = \begin{bmatrix} 1 & 3 & 4 & -1 & 2 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

6. If a 7×5 matrix A has rank 2, find dim Nul A, dim Row A, and rank A^T.

7. Suppose a 4×7 matrix A has four pivot columns. Is Col $A = \mathbb{R}^4$? Is Nul $A = \mathbb{R}^3$? Explain your answers.

12. If the null space of a 5×4 matrix A is 2-dimensional, what is the dimension of the row space of A?

In Exercises 17 and 18, A is an $m \times n$ matrix. Mark each statement True or False. Justify each answer.

17. a. The row space of A is the same as the column space of A^T.

b. If B is any echelon form of A, and if B has three nonzero rows, then the first three rows of A form a basis for Row A.

c. The dimensions of the row space and the column space of A are the same, even if A is not square.

d. The sum of the dimensions of the row space and the null space of A equals the number of rows in A.

22. Is is possible that all solutions of a homogeneous system of ten linear equations in twelve variables are multiples of one fixed nonzero solution? Discuss.