

Week 6 Homework Outline Solutions.

$$\begin{aligned}
 1) \det \begin{bmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{bmatrix} &= 3 \det \begin{bmatrix} 3 & 2 \\ 5 & -1 \end{bmatrix} + 4 \det \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} \\
 &= 3(-3-10) + 4(10-0) \\
 &= -39 + 40 = \underline{1}
 \end{aligned}$$

9) ~~det 3rd row:~~

$$\det A = 2 \det \begin{bmatrix} 0 & 0 & 5 \\ 7 & 2 & -5 \\ 3 & 1 & 8 \end{bmatrix} = 10 \det \begin{bmatrix} 7 & 2 \\ 3 & 1 \end{bmatrix} = 10(7-6) = \underline{10}$$

$$5) \det \begin{bmatrix} 1 & 5 & -6 \\ -1 & -4 & 4 \\ -2 & -7 & 9 \end{bmatrix} = \det \begin{bmatrix} 0 & 1 & -2 \\ -1 & -4 & 4 \\ 0 & 1 & 1 \end{bmatrix} = \det \begin{bmatrix} 1 & 4 & -4 \\ 0 & 1 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \det \begin{bmatrix} 1 & 4 & -4 \\ 0 & 1 & -2 \\ 0 & 0 & 3 \end{bmatrix} = 3 \times 1 \times 1 = \underline{3}$$

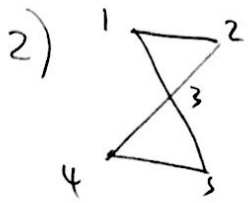
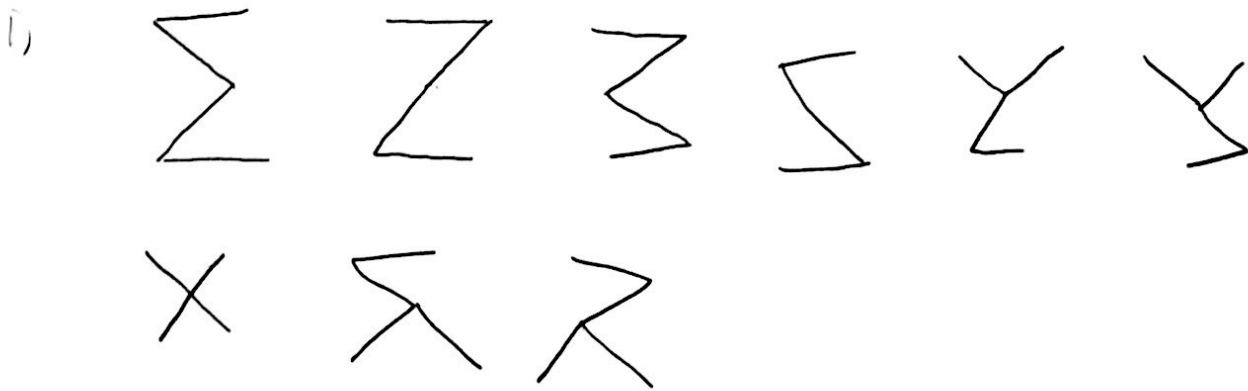
$$7) \det \begin{bmatrix} 1 & 3 & 0 & 2 \\ -2 & -5 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 1 & -1 & 2 & -3 \end{bmatrix} = \det \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7 & 8 \\ 0 & -4 & 2 & -5 \\ 0 & -4 & 2 & -5 \end{bmatrix} = \det \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7 & 8 \\ 0 & -4 & 2 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \underline{0}$$

$$19) \det \begin{bmatrix} 5 & 6 \\ 2 & 4 \end{bmatrix} = |20-12| = 8$$

$$20) \det \begin{bmatrix} -1 & 4 \\ 3 & -5 \end{bmatrix} = |5-12| = 7$$

Spanning Trees



Laplacian depends on ordering.

$$L = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ -1 & -1 & 4 & -1 & -1 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & -1 & 2 \end{bmatrix} \end{matrix}$$

$$\text{cof}_{11} L = (-1)^2 \det \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 4 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} = \det \begin{bmatrix} 0 & 7 & -2 & -2 \\ -1 & 4 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

$$= \det \begin{bmatrix} 7 & -2 & -2 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} = \det \begin{bmatrix} 7 & -2 & -2 \\ 0 & 3 & -3 \\ -1 & -1 & 2 \end{bmatrix}$$

$$= 3 \det \begin{bmatrix} 7 & -2 & -2 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \end{bmatrix} = 3 \left[7 \cdot (2-1) + (-1)(2+2) \right] = 3 \cdot 3 = 9 \quad \checkmark$$