Read throughout the suggested sections of the book. Attempt the questions below ready for the test on Thursday. Attempt the practise exam (see website) ready for Monday.

Find the inverses of the matrices in Exercises 1–4.

1. $\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}$

5. Use the inverse found in Exercise 1 to solve the system

$$\begin{aligned} 8x_1 + 6x_2 &= 2 \\ 5x_1 + 4x_2 &= -1 \end{aligned}$$

8. Suppose P is invertible and $A = PBP^{-1}$. Solve for B in terms of A.

16. Suppose A and B are $n \times n$ matrices, B is invertible, and AB is invertible. Show that A is invertible. [*Hint:* Let $C = AB$, and solve this equation for A.]

25. Show that if $ad - bc = 0$, then the equation $A\mathbf{x} = \mathbf{0}$ has more than one solution. Why does this imply that A is not invertible? [*Hint:* First, consider $a = b = 0$. Then, if a and b are not both zero, consider the vector $\mathbf{x} = \begin{bmatrix} -b \\ a \end{bmatrix}$.]

Find the inverses of the matrices in Exercises 29–32, if they exist.
Do this by row reducing the matrix [A:I_n].

29. $\begin{bmatrix} 1 & -3 \\ 4 & -9 \end{bmatrix}$

31. $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$

13. An $m \times n$ **upper triangular matrix** is one whose entries *below* the main diagonal are 0's (as in Exercise 8). When is a square upper triangular matrix invertible? Justify your answer.

In Exercises 11 and 12, the matrices are all $n \times n$. Each part of the exercises is an *implication* of the form "If $\langle$ statement 1 $\rangle$, then $\langle$ statement 2 $\rangle$." Mark an implication as True if the truth of $\langle$ statement 2 $\rangle$ *always* follows whenever $\langle$ statement 1 $\rangle$ happens to be true. An implication is False if there is an instance in which $\langle$ statement 2 $\rangle$ is false but $\langle$ statement 1 $\rangle$ is true. Justify each answer.

11. a. If the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then A is row equivalent to the $n \times n$ identity matrix.

b. If the columns of A span $\mathbb{R}^n$, then the columns are linearly independent.

c. If A is an $n \times n$ matrix, then the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$.

d. If the equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution, then A has fewer than n pivot positions.

e. If A^T is not invertible, then A is not invertible.

Compute the determinants in Exercises 1–8 using a cofactor expansion across the first row. In Exercises 1–4, also compute the determinant by a cofactor expansion down the second column.

1. $\begin{vmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{vmatrix}$

Compute the determinants in Exercises 9–14 by cofactor expansions. At each step, choose a row or column that involves the least amount of computation.

9. $\begin{vmatrix} 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \end{vmatrix}$

Find the determinants in Exercises 5–10 by row reduction to echelon form.

5. $\begin{vmatrix} 1 & 5 & -6 \\ -1 & -4 & 4 \\ -2 & -7 & 9 \end{vmatrix}$

7. $\begin{vmatrix} 1 & 3 & 0 & 2 \\ -2 & -5 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 1 & -1 & 2 & -3 \end{vmatrix}$

In Exercises 19–22, find the area of the parallelogram whose vertices are listed.

19. $(0, 0)$, $(5, 2)$, $(6, 4)$, $(11, 6)$

20. $(0, 0)$, $(-1, 3)$, $(4, -5)$, $(3, -2)$