Week 2 homework.

24. The system is consistent, see Theorem 2 (existence & uniqueness)

9 $x_1\begin{bmatrix}0\\4\\-1\end{bmatrix} + x_2\begin{bmatrix}1\\6\\3\end{bmatrix} + x_3\begin{bmatrix}5\\-1\\-8\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$.

10 $x_1\begin{bmatrix}3\\-2\\5\end{bmatrix} + x_2\begin{bmatrix}-2\\-7\\4\end{bmatrix} + x_3\begin{bmatrix}4\\5\\-3\end{bmatrix} = \begin{bmatrix}3\\1\\2\end{bmatrix}$.

11 Row reduction of A(M) is $\left[\begin{array}{ccc|c}1&0&5&2\\0&1&4&3\\0&0&0&0\end{array}\right]$.

Last column is not a pivot, so system is consistent, so $\underline{b}$ is a linear combination of the $\underline{a}_i$.

12 Row reduction of A(M) is $\left[\begin{array}{ccc|c}1&0&0&245/33\\0&1&0&-41/33\\0&0&1&-2/11\end{array}\right]$.

As for 11, $\underline{b}$ is a linear combination of the $\underline{a}_i$.

22 Many possibilities, eg. $A = \begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$, $\underline{b} = \begin{bmatrix}1\\0\\0\end{bmatrix}$.

1 Undefined - A is 3×2, and $\underline{b} \in \mathbb{R}^3$.
2 Undefined - A is 3×1, & $\underline{b} \in \mathbb{R}^2$.

8 Matrix equation:

$$\begin{bmatrix} 2 & -1 & -4 & 0 \\ -4 & 5 & 3 & 2 \end{bmatrix} \underline{x} = \begin{bmatrix} 5 \\ 12 \end{bmatrix} \qquad \left(\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \right)$$

14 Row reduce the matrix $\left[\begin{array}{ccc|c} 2 & 5 & -1 & 4 \\ 0 & 1 & -1 & -1 \\ 1 & 2 & 0 & 4 \end{array}\right]$

to get $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

System is <u>inconsistent</u>, so $\underline{u}$ is <u>not</u> a linear combination of the columns of A, so is <u>not</u> in the span.

24 a) True; if $A = [\underline{a}_1 \cdots \underline{a}_n]$ then the vector eq'n is

$$x_1 \underline{a}_1 + \cdots + x_n \underline{a}_n = \underline{b}.$$

b) True; ~~a~~ if $A = [\underline{a}_1, \ldots, \underline{a}_n]$, can write

$$\underline{b} = x_1 \underline{a}_1 + \cdots + x_n \underline{a}_n \quad \text{some } x_i \in \mathbb{R},$$

so $\underline{b}$ is a linear combination of the columns.

c) True; Put the vectors as the columns of A, & take $\underline{x}$ as the weights (coefficients) of the linear combinations.

d) False, eg $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\underline{b} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, then $A\underline{x} = \underline{b}$ is consistent.

e) True, See theorem 3.

f) False, eg take m = n = 1, A = [0], and b = [1]

30 $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. Every $\underline{v}$ in the span of the columns of A has a 0 in the final position.

(Many possibilities)

3 $\begin{bmatrix} 4 \\ 9 \\ 16 \end{bmatrix}$

4 $\begin{bmatrix} 3 \\ 8 \end{bmatrix}$.

Parametric vector form:
The augmented coefficient matrix is
[-4 -2 2 -4]
[3 1 4 4]

which row-reduces to
[1   0   5   2]
[0   1 -11 -2].
As equations, this becomes
x_1 + 5x_3 = 2
x_2 - 11x_3 = -2.
We rearrange these:
x_1 = 2 - 5x_3
x_2 = -2 + 11x_3
x_3 = 0 + 1x_3.
Where does the last equation come from? Nowhere, it's just always true! Whenever we have a free variable, we can always insert an equation like this. Reading off the columns in the above equations, we see that the parametric vector form of the general solution is

$$x = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} + s \begin{bmatrix} -5 \\ 11 \\ 1 \end{bmatrix}, \text{ s in R.}$$