

In Exercises 1–6, determine which sets of vectors are orthogonal.

$$1. \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -4 \\ -7 \end{bmatrix} \qquad 2. \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -5 \\ -2 \\ 1 \end{bmatrix}$$

In Exercises 7–10, show that $\{\mathbf{u}_1, \mathbf{u}_2\}$ or $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is an orthogonal basis for \mathbb{R}^2 or \mathbb{R}^3 , respectively. Then express \mathbf{x} as a linear combination of the \mathbf{u} 's.

$$7. \mathbf{u}_1 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 6 \\ 4 \end{bmatrix}, \text{ and } \mathbf{x} = \begin{bmatrix} 9 \\ -7 \end{bmatrix}$$

12. Compute the orthogonal projection of $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ onto the line through $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$ and the origin.

In Exercises 17–22, determine which sets of vectors are orthonormal. If a set is only orthogonal, normalize the vectors to produce an orthonormal set.

$$17. \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}, \begin{bmatrix} -1/2 \\ 0 \\ 1/2 \end{bmatrix} \qquad 18. \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

26. Suppose W is a subspace of \mathbb{R}^n spanned by n nonzero orthogonal vectors. Explain why $W = \mathbb{R}^n$.

In Exercises 3–6, verify that $\{\mathbf{u}_1, \mathbf{u}_2\}$ is an orthogonal set, and then find the orthogonal projection of \mathbf{y} onto $\text{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$.

$$3. \quad \mathbf{y} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

In Exercises 21 and 22, all vectors and subspaces are in \mathbb{R}^n . Mark each statement True or False. Justify each answer.

21. a. If \mathbf{z} is orthogonal to \mathbf{u}_1 and to \mathbf{u}_2 and if $W = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$, then \mathbf{z} must be in W^\perp .
- b. For each \mathbf{y} and each subspace W , the vector $\mathbf{y} - \text{proj}_W \mathbf{y}$ is orthogonal to W .
- c. The orthogonal projection $\hat{\mathbf{y}}$ of \mathbf{y} onto a subspace W can sometimes depend on the orthogonal basis for W used to compute $\hat{\mathbf{y}}$.
- d. If \mathbf{y} is in a subspace W , then the orthogonal projection of \mathbf{y} onto W is \mathbf{y} itself.

Find an orthogonal basis for the column space of each matrix in Exercises 9–12.

$$9. \quad \begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix}$$

