Universiteit Leiden
Wiskunde en Natuurwetenschappen

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① a)
$$\left(\begin{array}{cccc|c} -4 & 0 & 4 & 2 & 4 \\ 0 & 2 & 2 & -3 & 4 \\ 2 & 0 & -2 & 0 & 0 \end{array}\right)$$

b)
$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right)$$

c) Start by writing as equations:

$$\begin{aligned} x_1 \quad - x_3 \quad\quad &= 0 \\ x_2 + x_3 \quad\quad &= 5 \\ x_4 &= 2 \end{aligned}$$

free

$x_1 = x_3$
$x_2 = 5 - x_3$
$x_3 = \text{free}$
$x_4 = 2$

~~$x_3 = 0$: Gen soln: $x =$~~

eg. $x_3 = 0$: $\begin{bmatrix} 0 \\ 5 \\ 0 \\ 2 \end{bmatrix}$ $\quad x_3 = 1$: $\begin{bmatrix} 1 \\ 4 \\ 1 \\ 2 \end{bmatrix}$ $\quad x_3 = 2$: $\begin{bmatrix} 2 \\ 3 \\ 2 \\ 2 \end{bmatrix}$.

(2) a) $\det A = 1$.

b) One solution, since A is invertible.

c) $A^{-1} = \begin{bmatrix} -17 & -20 & -1 \\ 1 & 1 & 0 \\ -4 & -5 & 0 \end{bmatrix}$.

d) $\underline{x} = A^{-1}\underline{c} = \begin{bmatrix} -39 \\ 2 \\ -9 \end{bmatrix}$ Check that $A\underline{x} = \underline{c}$. ✓

3) a) Yes. Eg. $\det \begin{pmatrix} 1 & 3 \\ -5 & -5 \end{pmatrix} \neq 0$.

b) det ~~rowreduce~~
Yes. $\det \begin{bmatrix} 1 & 3 & 4 \\ -5 & -5 & 5 \\ 2 & 4 & 3 \end{bmatrix} = 0$, so must be a linear relation between columns, & first two are lin. indep, so $c \in \text{Span}(a, b)$.

c) No. If a set of n vectors spans $\mathbb{R}^n$ then it must be linearly independent.

4) $x_1 = 1$
$x_2 = 1$
$x_1 = x_2 + x_5$

$x_3 = 2$
$x_4 = 2$
$x_4 = x_3 + x_5$

b) ACM: $\left(\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right)$

Rowreduce to

$$\left(\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

(so unique solution, & flow $x_5 = 0$).

c) No. Let x_6 flow from X to Y. Then a solution is

$x_1 = 1$
$x_2 = 0$
$x_3 = 2$
$x_4 = 3$
$x_5 = 1$
$x_6 = 1$

5a) True, eg. given by ~~matrix~~ $\begin{bmatrix}x\\y\end{bmatrix} \mapsto \begin{bmatrix}x\\y\\0\end{bmatrix}$

b) True. Say $A\underline{0}=\underline{b}$, then $\underline{b}=0$,
so homogeneous.

c) True. $(A^T+B^T)_{ij} = A_{ji}+B_{ji} = (A+B)^T_{ij}$.

d) False. Eg. $G =$  , $\underline{n=2}$, $T=G$,
then T has $\underline{\text{one}}$ edge.

e) True. Adding copy of row 1 to row i for $i \neq 1$
does not change det.
Adding copy of row 1 to row 1 multiplies
det by 2.

6a) $\begin{pmatrix}3&6&9\\4&5&6\\7&8&9\end{pmatrix}$

b) $\begin{pmatrix}3&2&3\\12&5&6\\21&8&9\end{pmatrix}$

~~c)~~ $\cancel{\begin{pmatrix}1&3&2\\4&6&5\\7&9&8\end{pmatrix}}$

c ~~d~~) $\begin{pmatrix}1&2&3\\7&8&9\\4&5&6\end{pmatrix}$

d) $\begin{pmatrix}1&3&2\\4&6&5\\7&9&8\end{pmatrix}$

e) eg. $E_3 = \begin{pmatrix}1&0&0\\0&2&0\\0&0&1\end{pmatrix}$