Retake Exam – Lineaire Algebra 1

15 March 2019

Time: 3 hours.

Fill in your name and student number on all papers you hand in.

In total there are 6 questions, and each question is worth the same number of points.

In all questions, justify your answer fully and show all your work.

In this examination you are only allowed to use a pen and examination paper.

1. Consider the matrix A given by

$$A = \begin{bmatrix} -2 & 1 & -4 & 1 \\ -2 & 2 & -4 & 0 \\ -1 & -2 & -2 & 3 \end{bmatrix}$$

   a) Put the matrix A in reduced echelon form. *Hint: your answer should be one of the following two options:*

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

   b) Write the solution set of the homogeneous linear system $A\underline{x} = \underline{0}$ as a span of vectors.

2. Define matrices A and B by

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 1 & 0 & 2 \\ -1 & 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} -4 & 3 & -2 \\ 2 & 4 & 0 \\ 3 & -1 & 1 \end{bmatrix}$$

   a) Compute the inverse of A.

   b) Compute the determinant of B.

   c) Compute the product AB?

   d) What is the determinant of $-B$?

3. Let $\underline{u} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ and $\underline{v} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$. Let T be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$. Suppose

$$T(\underline{u}) = \begin{bmatrix} -3 \\ -3 \end{bmatrix} \text{ and } T(\underline{v}) = \begin{bmatrix} -11 \\ -5 \end{bmatrix}.$$

a) Compute $-3\underline{u} + \underline{v}$.

b) Use the linearity of T to compute $T(-3\underline{u} + \underline{v})$.

c) What is $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$?

d) Compute $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$. Hint: try to write $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ in terms of $\underline{u}$ and $\underline{v}$.

e) Write down the standard matrix for T.

4. In each part below, a function $\mathbb{R}^2 \to \mathbb{R}^2$ is given. For each part, state whether or not the function is *linear*. Justify your answers.

a) $f_a \colon \mathbb{R}^2 \to \mathbb{R}^2$ sends a vector $\underline{v}$ to $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \underline{v}$.

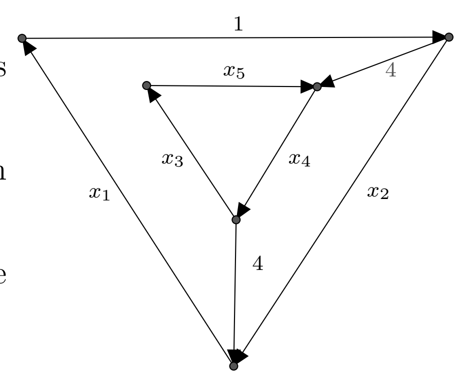
b) $f_b \colon \mathbb{R}^2 \to \mathbb{R}^2$ sends every vector $\underline{v}$ to the zero vector in $\mathbb{R}^2$.

c) $f_c \colon \mathbb{R}^2 \to \mathbb{R}^2$ sends $\begin{bmatrix} x \\ y \end{bmatrix}$ to $\begin{bmatrix} |x| \\ y \end{bmatrix}$, where $|x|$ denotes the absolute value of the real number x.

d) $f_d = f_a \circ f_c$ (composite function).

e) $f_e = f_c \circ f_b$ (composite function).

5. Consider the network on the right.



a) Write down a linear system describing the flow in this network.

b) Put the augmented matrix of the linear system from (a) in row reduced echelon form.

c) Does there exist a solution such that the flow along the edge labelled x_3 is greater than 100?

6. For each of the following 5 statements, say whether the statement is true or false. Justify your answer (either by an example if the statement is false, or a brief proof if it is true). If you are unsure, try some small examples.

   a) If A and B are invertible $n \times n$ matrices, then $A + B$ is also invertible.

   b) If all the entries of a 2×2 matrix are positive (> 0), then the determinant of the matrix is positive (> 0).

   c) Let A be an $n \times n$ matrix with $n \geq q$. If A has two rows equal then $\det A = 0$.

   d) If a linear system has zero as a solution then it must be homogeneous.

   e) If A is an $n \times n$ matrix and A^2 is invertible, then A is invertible.