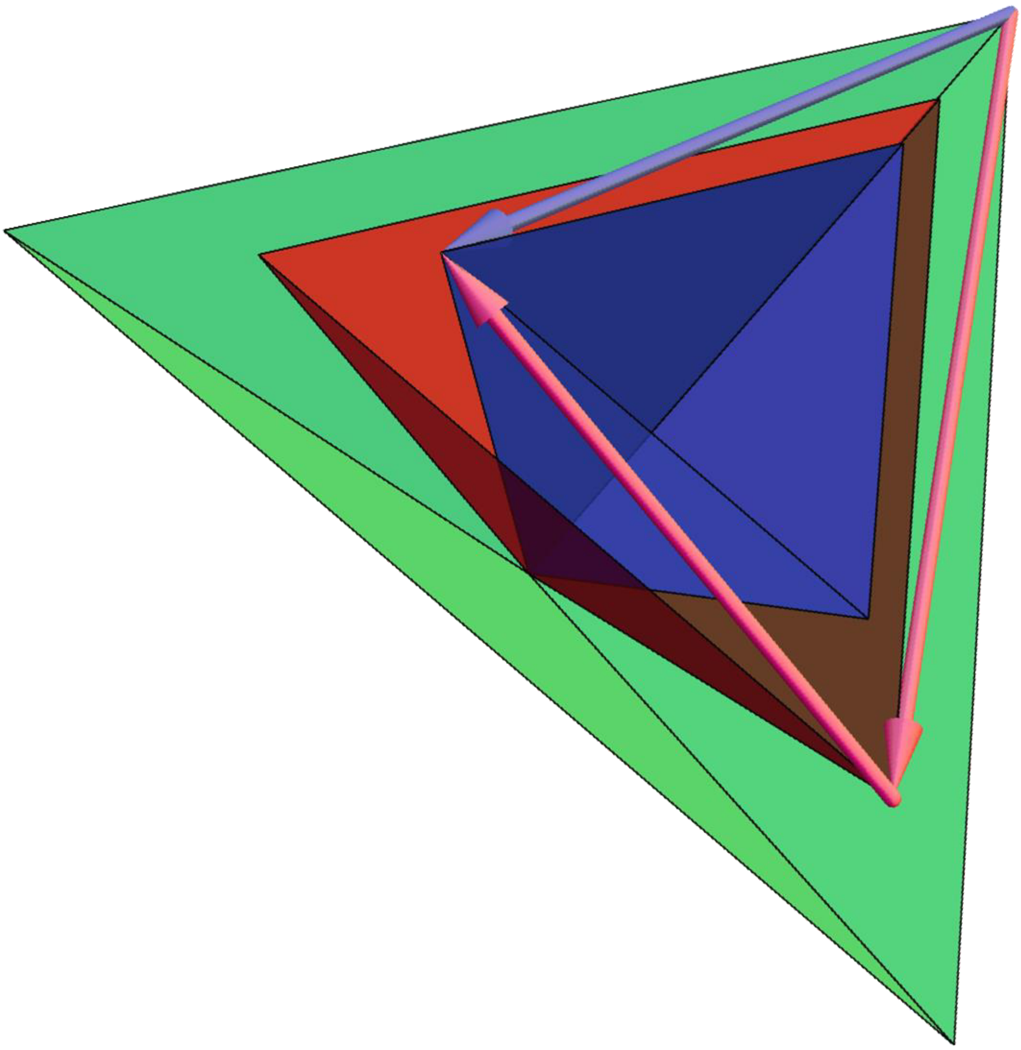
Bibliography on Ordered Banach Algebras



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Bibliography on Ordered Banach Algebras

[1] Y. A. Abramovich, E. L. Arenson, and A. K. Kitover, *Banach $C(K)$-modules and operators preserving disjointness*, Pitman Research Notes in Mathematics Series, vol. 277, Longman Scientific & Technical, Harlow, 1992. MR1202880 (94d:47027)

[2] P. Aiena and U. Oliveri, *Pure states in ordered locally convex Hausdorff spaces*, Rend. Circ. Mat. Palermo (2) **29** (1980), no. 3, 427–434 (1981), DOI 10.1007/BF02849759. MR638681 (83a:46014)

[3] Egor A. Alekhno, *Resolvent of a positive element of an ordered Banach algebra*, Proceedings of the 5th Int. Conference "AMADE" (Minsk, 2009), Vol. 1, 2010, pp. 19–24 (Russian).

[4] ———, *Hereditary properties of dominated elements*, Proceedings of the 6th Int. Conference "AMADE" (Minsk, 2011), Vol. 1, 2012, pp. 23–27 (Russian).

[5] ———, *The irreducibility in ordered Banach algebras*, Positivity **16** (2012), no. 1, 143–176, DOI 10.1007/s11117-011-0117-9. MR2892579

[6] ———, *On ordered C^*-algebras*, (submitted).

[7] Charalambos D. Aliprantis and Owen Burkinshaw, *Positive compact operators on Banach lattices*, Math. Z. **174** (1980), no. 3, 289–298, DOI 10.1007/BF01161416. MR593826 (81m:47053)

[8] F. Aràndiga and V. Caselles, *Approximations of positive operators and continuity of the spectral radius*, J. Operator Theory **26** (1991), no. 1, 53–71. MR1214920 (94d:47035)

[9] ———, *Approximations of positive operators and continuity of the spectral radius. II*, Math. Z. **209** (1992), no. 4, 547–558, DOI 10.1007/BF02570853. MR1156436 (94d:47036)

[10] ———, *Approximations of positive operators and continuity of the spectral radius. III*, J. Austral. Math. Soc. Ser. A **57** (1994), no. 3, 330–340. MR1297007 (95j:47045)

[11] Wolfgang Arendt, *On the* o*-spectrum of regular operators and the spectrum of measures*, Math. Z. **178** (1981), no. 2, 271–287, DOI 10.1007/BF01262044. MR631633 (83h:47027)

[12] Wolfgang Arendt and Hans-Ulrich Schwarz, *Ideale regulärer Operatoren und Kompaktheit positiver Operatoren zwischen Banachverbänden*, Math. Nachr. **131** (1987), 7–18, DOI 10.1002/mana.19871310102 (German). MR908795 (89g:47048)

[13] W. Arendt and A. R. Sourour, *Perturbation of regular operators and the order essential spectrum*, Nederl. Akad. Wetensch. Indag. Math. **48** (1986), no. 2, 109–122. MR849713 (87j:47052)

[14] ———, *Ideals of regular operators on l^2*, Proc. Amer. Math. Soc. **88** (1983), no. 1, 93–96, DOI 10.2307/2045116. MR691284 (84c:47048)

[15] Mohamed Basly, *Produit naturel pour les FF-algèbres de Banach réticulées de type $C(X)$*, Funct. Approx. Comment. Math. **18** (1989), 63–66 (French, with English summary). MR1020394 (91f:46072)

[16] D. Behrendt and H. Raubenheimer, *On domination of inessential elements in ordered Banach algebras*, Illinois J. Math. **51** (2007), no. 3, 927–936. MR2379731 (2009b:46102)

[17] S. J. Bernau and C. B. Huijsmans, *On the positivity of the unit element in a normed lattice ordered algebra*, Studia Math. **97** (1990), no. 2, 143–149. MR1083344 (92d:46054)

[18] O. Blasco, H.G. Dales, and H.L. Pham, *Equivalences involving (p;q)-multi-norms*, (submitted).

[19] Gareth Braatvedt, Rudi Brits, and Heinrich Raubenheimer, *Gelfand-Hille type theorems in ordered Banach algebras*, Positivity **13** (2009), no. 1, 39–50, DOI 10.1007/s11117-008-2200-4. MR2466228 (2010a:46107)

[20] G. Brown, *Type* 0 *semi-algebras in Banach algebras*, J. London Math. Soc. **43** (1968), 482–486. MR0234285 (38 #2602)

[21] ———, *Norm properties of a class of semi-algebras*, J. London Math. Soc. **44** (1969), 329–339. MR0244767 (39 #6081)

[22] I. Burger and J. J. Grobler, *Spectral properties of positive elements in Banach lattice algebras*, Quaestiones Math. **18** (1995), no. 1-3, 261–270. First International Conference in Abstract Algebra (Kruger Park, 1993). MR1340482 (96h:46069)

[23] Laura Burlando, *Monotonicity of spectral radius for positive operators on ordered Banach spaces*, Arch. Math. (Basel) **56** (1991), no. 1, 49–57, DOI 10.1007/BF01190081. MR1082002 (91k:47083)

[24] V. Caselles, *On irreducible operators on Banach lattices*, Nederl. Akad. Wetensch. Indag. Math. **48** (1986), no. 1, 11–16. MR834316 (87i:47050)

[25] ———, *On the peripheral spectrum of positive operators*, Israel J. Math. **58** (1987), no. 2, 144–160, DOI 10.1007/BF02785673. MR901175 (88j:47054)

[26] V. B. Cherdak, *On the order spectrum of r-compact operators in lattice-normed spaces*, Sibirsk. Mat. Zh. **32** (1991), no. 1, 148–152, 221, DOI 10.1007/BF00970168 (Russian); English transl., Siberian Math. J. **32** (1991), no. 1, 123–126. MR1112090 (92h:47006)

[27] Ion Chiţescu, *Köthe spaces that are Banach algebras with unit*, Bull. Math. Soc. Sci. Math. R. S. Roumanie (N.S.) **18(66)** (1974), no. 3-4, 269–271 (1976). MR0458153 (56 #16356)

[28] Stephen T. L. Choy, *Positive operators and algebras of dominated measures*, Rev. Roumaine Math. Pures Appl. **34** (1989), no. 3, 213–219. MR1006639 (90g:47055)

[29] Taen Yu Dai and Ralph DeMarr, *Positive derivations on partially ordered linear algebra with an order unit*, Proc. Amer. Math. Soc. **72** (1978), no. 1, 21–26. MR0491402 (58 #10658)

[30] H.G. Dales, *Multi-norms*, (Talk given in Tartu in 5 September 2013).

[31] H. G. Dales, M. Daws, H. L. Pham, and P. Ramsden, *Multi-norms and the injectivity of $L^p(G)$*, J. Lond. Math. Soc. (2) **86** (2012), no. 3, 779–809, DOI 10.1112/jlms/jds026. MR3000830

[32] H. G. Dales, Tomasz Kania, Tomasz Kochanek, Piotr Koszmider, and Niels Jakob Laustsen, *Maximal left ideals of the Banach algebra of bounded operators on a Banach space*, Studia Math. **218** (2013), no. 3, 245–286, DOI 10.4064/sm218-3-3. MR3125128

[33] H. G. Dales, A. T.-M. Lau, and D. Strauss, *Second duals of measure algebras*, Dissertationes Math. (Rozprawy Mat.) **481** (2012), 1–121, DOI 10.4064/dm481-0-1. MR2920625

[34] H. G. Dales and M. E. Polyakov, *Multi-normed spaces*, Dissertationes Math. (Rozprawy Mat.) **488** (2012), 165, DOI 10.4064/dm488-0-1. MR3024929

[35] B. C. Dhage, *Fixed point theorems in ordered Banach algebras and applications*, Panamer. Math. J. **9** (1999), no. 4, 83–102. MR1724533 (2000i:47104)

[36] Sjoerd Dirksen, Marcel de Jeu, and Marten Wortel, *Extending representations of normed algebras in Banach spaces*, Operator structures and dynamical systems, Contemp. Math., vol. 503, Amer. Math. Soc., Providence, RI, 2009, pp. 53–72, DOI 10.1090/conm/503/09893, (to appear in print). MR2590616 (2011a:46072)

[37] P. G. Dixon, *Topologically nilpotent Banach algebras and factorisation*, Proc. Roy. Soc. Edinburgh Sect. A **119** (1991), no. 3-4, 329–341, DOI 10.1017/S0308210500014876. MR1135978 (93d:46073)

[38] P. G. Dodds and D. H. Fremlin, *Compact operators in Banach lattices*, Israel J. Math. **34** (1979), no. 4, 287–320 (1980), DOI 10.1007/BF02760610. MR570888 (81g:47037)

[39] Roman Drnovšek, *Triangularizing semigroups of positive operators on an atomic normed Riesz space*, Proc. Edinburgh Math. Soc. (2) **43** (2000), no. 1, 43–55, DOI 10.1017/S001309150002068X. MR1744698 (2001i:47062)

[40] ———, *Common invariant subspaces for collections of operators*, Integral Equations Operator Theory **39** (2001), no. 3, 253–266, DOI 10.1007/BF01332655. MR1818060 (2001m:47012)

[41] H. A. Dye and R. S. Phillips, *Groups of positive operators*, Canad. J. Math. **8** (1956), 462–486. MR0087906 (19,434a)

[42] A. R. Esajan, *On estimating the spectrum of the sum of positive semi-commuting operators*, Sibirsk. Mat. Ž. **7** (1966), 460–464 (Russian). MR0194893 (33 #3099)

[43] Günther Greiner, *A spectral decomposition of strongly continuous groups of positive operators*, Quart. J. Math. Oxford Ser. (2) **35** (1984), no. 137, 37–47, DOI 10.1093/qmath/35.1.37. MR734664 (85i:47038)

[44] G. Greiner and U. Groh, *A Perron Frobenius theory for representations of locally compact abelian groups*, Math. Ann. **262** (1983), no. 4, 517–528, DOI 10.1007/BF01456066. MR696523 (84f:47046)

[45] J. J. Grobler, *The zero-two law in Banach lattice algebras*, Israel J. Math. **64** (1988), no. 1, 32–38, DOI 10.1007/BF02767367. MR981746 (90e:46037)

[46] ———, *The zero-two law and the peripheral spectrum of positive contractions in Banach lattice algebras*, Operator theory: operator algebras and applications, Part 2 (Durham, NH, 1988), Proc. Sympos. Pure Math., vol. 51, Amer. Math. Soc., Providence, RI, 1990, pp. 135–141. MR1077430

[47] J. J. Grobler and C. B. Huijsmans, *Doubly Abel bounded operators with single spectrum*, Quaestiones Math. **18** (1995), no. 4, 397–406. MR1354120 (96i:47001)

[48] Emilie Haynsworth and J. R. Wall, *Group inverses of certain nonnegative matrices*, Linear Algebra Appl. **25** (1979), 271–288, DOI 10.1016/0024-3795(79)90023-5. MR528730 (80b:15006)

[49] ———, *Group inverses of certain positive operators*, Linear Algebra Appl. **40** (1981), 143–159, DOI 10.1016/0024-3795(81)90146-4. MR629613 (82j:15004)

[50] Gerd Herzog, *Generalized M-matrices and ordered Banach algebras*, Linear Algebra Appl. **363** (2003), 125–131, DOI 10.1016/S0024-3795(02)00552-9.

Special issue on nonnegative matrices, M-matrices and their generalizations (Oberwolfach, 2000). MR1969064 (2004b:15041)

[51] Gerd Herzog and Roland Lemmert, *On quasipositive elements in ordered Banach algebras*, Studia Math. **129** (1998), no. 1, 59–65. MR1611855 (99g:46061)

[52] ______, *On Riccati equations in ordered Banach algebras*, Demonstratio Math. **35** (2002), no. 4, 783–790. MR1941348 (2003j:34107)

[53] Gerd Herzog and Christoph Schmoeger, *A note on a theorem of Raubenheimer and Rode*, Proc. Amer. Math. Soc. **131** (2003), no. 11, 3507–3509 (electronic), DOI 10.1090/S0002-9939-03-06915-6. MR1991762 (2004e:46054)

[54] ______, *An example on ordered Banach algebras*, Proc. Amer. Math. Soc. **135** (2007), no. 12, 3949–3954 (electronic), DOI 10.1090/S0002-9939-07-09000-4. MR2341945 (2008g:46071)

[55] ______, *Suprema of chains of operators*, Positivity **15** (2011), no. 2, 343–349, DOI 10.1007/s11117-010-0079-3. MR2803823 (2012m:47116)

[56] C. B. Huijsmans, *An inequality in complex Riesz algebras*, Studia Sci. Math. Hungar. **20** (1985), no. 1-4, 29–32. MR886000 (88i:46010)

[57] ______, *Elements with unit spectrum in a Banach lattice algebra*, Nederl. Akad. Wetensch. Indag. Math. **50** (1988), no. 1, 43–51. MR934473 (89f:46106)

[58] ______, *A characterization of complex lattice homomorphisms on Banach lattice algebras*, Quaestiones Math. **18** (1995), no. 1-3, 131–140. First International Conference in Abstract Algebra (Kruger Park, 1993). MR1340473 (96g:46013)

[59] C. B. Huijsmans and B. de Pagter, *Averaging operators and positive contractive projections*, J. Math. Anal. Appl. **113** (1986), no. 1, 163–184, DOI 10.1016/0022-247X(86)90340-9. MR826666 (87f:47055)

[60] I. A. Itskovich, *Linear models of optimization in real Banach algebras admitting of natural ordering*, Models and methods of investigating economic systems (Russian), "Nauka" Sibirsk. Otdel., Novosibirsk, 1979, pp. 60–90 (Russian). MR566945 (83a:90182)

[61] ______, *The von Neumann model in ordered Banach algebras*, Mathematical methods for the analysis of the interaction of branch and regional systems (Russian), Mat. Anal. Èkonom. Modeleĭ, "Nauka" Sibirsk. Otdel., Novosibirsk, 1983, pp. 44–50 (Russian). MR992638 (90b:90039)

[62] M. de Jeu and Messerschmidt. M., *Crossed products of Banach algebras. III*, Dissertationes Math. (Rozprawy Mat.), to appear.

[63] Marcel de Jeu and Marten Wortel, *Positive representations of finite groups in Riesz spaces*, Internat. J. Math. **23** (2012), no. 7, 1250076, 28, DOI 10.1142/S0129167X12500760. MR2945655

[64] ______, *Compact groups of positive operators on Banach lattices*, Indag. Math. (N.S.) **25** (2014), no. 2, 186–205, DOI 10.1016/j.indag.2012.05.003. MR3151814

[65] A. Jiménez-Vargas and Moisés Villegas-Vallecillos, *Order isomorphisms of little Lipschitz algebras*, Houston J. Math. **34** (2008), no. 4, 1185–1195. MR2465374 (2009k:46047)

[66] Richard V. Kadison, *A representation theory for commutative topological algebra*, Mem. Amer. Math. Soc., **1951** (1951), no. 7, 39. MR0044040 (13,360b)

[67] J. L. Kelley and R. L. Vaught, *The positive cone in Banach algebras*, Trans. Amer. Math. Soc. **74** (1953), 44–55. MR0054175 (14,883e)

[68] J.-L. Krivine, *Anneaux préordonnés*, J. Analyse Math. **12** (1964), 307–326 (French). MR0175937 (31 #213)

[69] M. Lashkarizadeh Bami, *Generalized positive definite functions and completely monotone functions on foundation semigroups*, J. Sci. Islam. Repub. Iran **11** (2000), no. 3, 245–252. MR1825056 (2002k:43003)

[70] Gordon MacDonald and Heydar Radjavi, *Standard triangularization of semigroups of non-negative operators*, J. Funct. Anal. **219** (2005), no. 1, 161–176, DOI 10.1016/j.jfa.2004.04.002. MR2108364 (2005m:47086)

[71] Laura Martignon, *Banach f-algebras and Banach lattice algebras with unit*, Bol. Soc. Brasil. Mat. **11** (1980), no. 1, 11–17, DOI 10.1007/BF02584875. MR607012 (82c:46027)

[72] ———, *Banach lattice algebras with multiplicative modulus and convolution Banach algebras*, II (Poços de Caldas, 1979), Cons. Nac. Desenvolvimento Ci. Tec., Inst. Mat. Pura Apl., Rio de Janeiro, 1981, pp. 237–248. MR689184 (84k:46041)

[73] Josep Martínez and José M. Mazón, *Quasi-compactness of dominated positive operators and C_0-semigroups*, Math. Z. **207** (1991), no. 1, 109–120, DOI 10.1007/BF02571379. MR1106817 (92g:47048)

[74] P. T. N. McPolin and A. W. Wickstead, *Relatively central operators on Archimedean vector lattices. III*, Quart. J. Math. Oxford Ser. (2) **36** (1985), no. 141, 75–89, DOI 10.1093/qmath/36.1.75. MR780352 (87g:47076)

[75] ———, *Relatively central operators on Archimedean vector lattices. II*, J. Austral. Math. Soc. Ser. A **40** (1986), no. 3, 287–298. MR832971 (87g:47075)

[76] John Boris Miller, *The natural ordering on a strictly real Banach algebra*, Math. Proc. Cambridge Philos. Soc. **107** (1990), no. 3, 539–556, DOI 10.1017/S0305004100068808. MR1041484 (91e:46060)

[77] H. du T. Mouton and S. Mouton, *Domination properties in ordered Banach algebras*, Studia Math. **149** (2002), no. 1, 63–73, DOI 10.4064/sm149-1-4. MR1881716 (2003e:46076)

[78] S. Mouton, *Convergence properties of positive elements in Banach algebras*, Math. Proc. R. Ir. Acad. **102A** (2002), no. 2, 149–162, DOI 10.3318/PRIA.2002.102.2.149. MR1961634 (2004d:47074)

[79] ———, *A spectral problem in ordered Banach algebras*, Bull. Austral. Math. Soc. **67** (2003), no. 1, 131–144, DOI 10.1017/S0004972700033591. MR1962967 (2004d:47075)

[80] ———, *On the boundary spectrum in Banach algebras*, Bull. Austral. Math. Soc. **74** (2006), no. 2, 239–246, DOI 10.1017/S0004972700035681. MR2260492 (2007g:46072)

[81] ———, *On spectral continuity of positive elements*, Studia Math. **174** (2006), no. 1, 75–84, DOI 10.4064/sm174-1-6. MR2239814 (2007c:46047)

[82] ———, *A condition for spectral continuity of positive elements*, Proc. Amer. Math. Soc. **137** (2009), no. 5, 1777–1782, DOI 10.1090/S0002-9939-08-09715-3. MR2470837 (2009m:46072)

[83] S. Mouton and K. Muzundu, *Commutatively ordered Banach algebras*, Quaestiones Mathematicae **36** (2013), no. 4, DOI 10.2989/16073606.2013.779985.

[84] ———, *Domination by ergodic elements in ordered Banach algebras*, Positivity **18** (2014), no. 1, 119–130, DOI 10.1007/s11117-013-0234-8. MR3167069
[85] S. Mouton and H. Raubenheimer, *More spectral theory in ordered Banach algebras*, Positivity **1** (1997), no. 4, 305–317, DOI 10.1023/A:1009717500980. MR1660397 (2000a:46070)
[86] K. Muzundu, *On poles of the resolvents of dominated elements in ordered Banach algebras*, Proceedings of Positivity VII, (to be published by Birkhauser).
[87] Kung-fu Ng, *A representation theorem for partially ordered Banach algebras*, Proc. Cambridge Philos. Soc. **64** (1968), 53–59. MR0220061 (36 #3128)
[88] B. de Pagter and W. J. Ricker, *Bicommutants of algebras of multiplication operators*, Proc. London Math. Soc. (3) **72** (1996), no. 2, 458–480, DOI 10.1112/plms/s3-72.2.458. MR1367086 (97a:47068)
[89] ———, *Algebras of multiplication operators in Banach function spaces*, J. Operator Theory **42** (1999), no. 2, 245–267. MR1716969 (2000j:47120)
[90] ———, *$C(K)$-representations and R-boundedness*, J. Lond. Math. Soc. (2) **76** (2007), no. 2, 498–512, DOI 10.1112/jlms/jdm072. MR2363429 (2009e:46037)
[91] B. de Pagter and A. R. Schep, *Measures of noncompactness of operators in Banach lattices*, J. Funct. Anal. **78** (1988), no. 1, 31–55, DOI 10.1016/0022-1236(88)90131-0. MR937631 (89d:47079)
[92] Frank Räbiger and Manfred P. H. Wolff, *Spectral and asymptotic properties of dominated operators*, J. Austral. Math. Soc. Ser. A **63** (1997), no. 1, 16–31. MR1456587 (98k:47073)
[93] ———, *Spectral and asymptotic properties of resolvent-dominated operators*, J. Austral. Math. Soc. Ser. A **68** (2000), no. 2, 181–201. MR1738035 (2001b:47009)
[94] Heinrich Raubenheimer, *The o-spectrum of r-asymptotically quasifinite-rank operators*, Quaestiones Math. **7** (1984), no. 3, 299–303. MR771655 (86c:47050)
[95] ———, *r-asymptotically quasifinite rank operators and the spectrum of measures*, Quaestiones Math. **10** (1986), no. 1, 97–111. MR857772 (87m:47090)
[96] ———, *On regular Riesz operators*, Quaest. Math. **23** (2000), no. 2, 179–186, DOI 10.2989/16073600009485966. MR1795734 (2002e:47021)
[97] H. Raubenheimer and S. Rode, *Cones in Banach algebras*, Indag. Math. (N.S.) **7** (1996), no. 4, 489–502, DOI 10.1016/S0019-3577(97)89135-5. MR1620116 (99i:46035)
[98] H. Render, *Lattice structures of ordered Banach algebras*, Illinois J. Math. **36** (1992), no. 2, 238–250. MR1156625 (93c:46095)
[99] Luis A. Sanchez, *Convex cones associated to generalized cones in $\mathbb{R}^N$*, Linear Algebra Appl. **433** (2010), no. 11-12, 2122–2138, DOI 10.1016/j.laa.2010.07.004. MR2736139 (2012c:15069)
[100] Karen Saxe, *Essential order spectra of convolution operators*, Indag. Math. (N.S.) **4** (1993), no. 1, 79–89, DOI 10.1016/0019-3577(93)90054-3. MR1213325 (94e:47045)
[101] H. H. Schaefer, *Some spectral properties of positive linear operators*, Pacific J. Math. **10** (1960), 1009–1019. MR0115090 (22 #5893)
[102] ———, *Invariant ideals of positive operators in $C(X)$. I*, Illinois J. Math. **11** (1967), 703–715. MR0218912 (36 #1996)

[103] ———, *Invariant ideals of positive operators in $C(X)$. II*, Illinois J. Math. **12** (1968), 525–538. MR0233216 (38 #1539)
[104] ———, *On the* o*-spectrum of order bounded operators*, Math. Z. **154** (1977), no. 1, 79–84. MR0470748 (57 #10494)
[105] ———, *Ordnungsstrukturen in der Operatorentheorie*, Jahresber. Deutsch. Math.-Verein. **82** (1980), no. 1, 33–50 (German). MR565135 (81f:47038)
[106] ———, *Some recent results on positive groups and semigroups*, From A to Z (Leiden, 1982), Math. Centre Tracts, vol. 149, Math. Centrum, Amsterdam, 1982, pp. 69–79. MR666257 (84j:47071)
[107] Helmut H. Schaefer, Manfred Wolff, and Wolfgang Arendt, *On lattice isomorphisms with positive real spectrum and groups of positive operators*, Math. Z. **164** (1978), no. 2, 115–123, DOI 10.1007/BF01174818. MR517148 (80b:47048)
[108] Egon Scheffold, *Das Spektrum von Verbandsoperatoren in Banachverbänden*, Math. Z. **123** (1971), 177–190 (German). MR0324475 (48 #2827)
[109] ———, *Über komplexe Banachverbandsalgebren*, J. Funct. Anal. **37** (1980), no. 3, 382–400, DOI 10.1016/0022-1236(80)90050-6 (German, with English summary). MR581429 (81m:46064)
[110] ———, *FF-Banachverbandsalgebren*, Math. Z. **177** (1981), no. 2, 193–205, DOI 10.1007/BF01214199 (German). MR612873 (82f:46060)
[111] ———, *Banachverbandsalgebren mit einer natürlichen Wedderburn-Zerlegung*, Math. Z. **185** (1984), no. 4, 521–531, DOI 10.1007/BF01236261 (German). MR733772 (85h:46070)
[112] ———, *Über den Spektralradius bei kommutativen Banachverbandsalgebren*, Arch. Math. (Basel) **44** (1985), no. 4, 365–368, DOI 10.1007/BF01235781 (German). MR788952 (86i:46056)
[113] ———, *Über Banachverbandsalgebren mit multiplikativer Zerlegungseigenschaft*, Acta Math. Hungar. **52** (1988), no. 3-4, 273–289, DOI 10.1007/BF01951573 (German). MR991863 (90i:46090)
[114] ———, *Über FF-Banachverbandsalgebren $C(K)$ und ihren assoziierten Operator*, Rev. Roumaine Math. Pures Appl. **34** (1989), no. 4, 347–354 (German, with English summary). MR1006654 (90f:46046)
[115] ———, *Der Bidual von F-Banachverbandsalgebren*, Acta Sci. Math. (Szeged) **55** (1991), no. 1-2, 167–179 (German). MR1124955 (92h:46027)
[116] ———, *Über Reynoldsoperatoren und "Mittelwert bildende" Operatoren auf halbeinfachen F-Banachverbandsalgebren*, Math. Nachr. **162** (1993), 329–337 (German). MR1239593 (95c:47042)
[117] ———, *Über den ordnungsstetigen Bidual von FF-Banachverbandsalgebren*, Arch. Math. (Basel) **60** (1993), no. 5, 473–477, DOI 10.1007/BF01202314 (German). MR1213518 (94g:46052)
[118] ———, *Über Bimorphismen und das Arens-Produkt bei kommutativen D-Banachverbandsalgebren*, Rev. Roumaine Math. Pures Appl. **39** (1994), no. 3, 259–270 (German, with English summary). MR1315492 (96d:46071)
[119] ———, *Maßalgebren mit intervallerhaltender linksseitiger Multiplikation*, Acta Math. Hungar. **76** (1997), no. 1-2, 59–67, DOI 10.1007/BF02907052 (German). MR1459770 (2000b:46079)
[120] ———, *Über Banachverbandsalgebren vom Typ 1*, Ark. Mat. **41** (2003), no. 2, 375–379, DOI 10.1007/BF02390821 (German). MR2011927 (2004h:46052)

[121] E. Scheffold, *Über positive Resolventenwerte positiver Operatoren*, Positivity **8** (2004), no. 2, 179–186, DOI 10.1023/B:POST.0000042834.68084.16 (German, with English summary). MR2097087 (2005i:47062)

[122] Anton R. Schep, *A remark on the uniform zero-two law for positive contractions*, Arch. Math. (Basel) **53** (1989), no. 5, 493–496, DOI 10.1007/BF01324724. MR1019163 (90j:47046)

[123] H. Schneider and R. E. L. Turner, *Positive eigenvectors of order-preserving maps*, J. Math. Anal. Appl. **37** (1972), 506–515. MR0288557 (44 #5754)

[124] Thomas I. Seidman and Hans Schneider, *The spectral radius in partially ordered algebras*, Linear Algebra Appl. **417** (2006), no. 2-3, 347–369, DOI 10.1016/j.laa.2005.10.018. MR2250317 (2007m:47005)

[125] A. R. Sourour, *Spectrum-preserving linear maps on the algebra of regular operators*, Aspects of positivity in functional analysis (Tübingen, 1985), North-Holland Math. Stud., vol. 122, North-Holland, Amsterdam, 1986, pp. 255–259. MR859735 (88d:47049)

[126] M. D. Srinivas, *Quantum generalization of Kolmogorov entropy*, J. Math. Phys. **19** (1978), no. 9, 1952–1961, DOI 10.1063/1.523916. MR502514 (80a:81030)

[127] V. Ja. Stecenko, *On a spectral property of an indecomposable operator*, Uspehi Mat. Nauk **22** (1967), 242–244 (Russian). MR0215116 (35 #5959)

[128] J. Synnatzschke, *Über einige verbandstheoretische Eigenschaften der Multiplikation von Operatoren in Vektorverbänden*, Math. Nachr. **95** (1980), 273–292, DOI 10.1002/mana.19800950125 (German). MR592901 (82b:47048)

[129] Joseph L. Taylor, *Noncommutative convolution measure algebras*, Pacific J. Math. **31** (1969), 809–826. MR0256187 (41 #844)

[130] A. C. Thompson and M. S. Vijayakumar, *An order-preserving representation theorem for complex Banach algebras and some examples*, Glasgow Math. J. **14** (1973), 128–135. MR0336344 (49 #1119)

[131] Nedra Toumi, *On some order ideals in Banach almost f-algebras*, Int. Math. Forum **3** (2008), no. 1-4, 107–113. MR2373493 (2008m:46042)

[132] Abdelmajid Triki, *Extensions of positive projections and averaging operators*, J. Math. Anal. Appl. **153** (1990), no. 2, 486–496, DOI 10.1016/0022-247X(90)90227-7. MR1080661 (91j:47043)

[133] Ayşe Uyar, *On Banach lattice algebras*, Turkish J. Math. **29** (2005), no. 3, 287–290. MR2160427 (2006d:46019)

[134] Lucas Venter, *A multiplication inequality in complex Banach lattice algebras*, Quaestiones Math. **8** (1985), no. 3, 275–281. MR833906 (87h:46111)

[135] Lucas Venter, Jacobus J. Grobler, and Peter van Eldik, *The peripheral spectrum in Banach lattice algebras*, Quaestiones Math. **12** (1989), no. 2, 175–185. MR998747 (90h:46084)

[136] A. M. Vershik, *Measurable realizations of automorphism groups and integral representations of positive operators*, Sibirsk. Mat. Zh. **28** (1987), no. 1, i, 52–60 (Russian). MR886852 (89i:22011)

[137] Lutz Weis, *An extrapolation theorem for the* 0*-spectrum*, Aspects of positivity in functional analysis (Tübingen, 1985), North-Holland Math. Stud., vol. 122, North-Holland, Amsterdam, 1986, pp. 261–269. MR859736 (87k:47075)

[138] A. J. White, *Ordered Banach algebras*, J. London Math. Soc. (2) **11** (1975), no. 2, 175–178. MR0399866 (53 #3708)

[139] A. W. Wickstead, *The structure space and ideal centre of a Banach lattice*, 1976. (Unpublished slides of a talk given at Silivri, Turkey, in 1976.)
[140] ______, *The ideal centre of a Banach lattice*, Proc. Roy. Irish Acad. Sect. A **76** (1976), no. 4, 15–23. MR0420214 (54 #8228)
[141] ______, *The structure space of a Banach lattice*, J. Math. Pures Appl. (9) **56** (1977), no. 1, 39–54. MR0477671 (57 #17188a)
[142] ______, *The structure space of a Banach lattice. II*, Proc. Roy. Irish Acad. Sect. A **77** (1977), no. 9, 105–111. MR0477672 (57 #17188b)
[143] ______, *Relatively central operators on Archimedean vector lattices. I*, Proc. Roy. Irish Acad. Sect. A **80** (1980), no. 2, 191–208. MR620620 (82g:47027)
[144] ______, *An embedding of the algebra of order bounded operators on a Dedekind complete Banach lattice*, Math. Z. **208** (1991), no. 1, 161–166, DOI 10.1007/BF02571517. MR1125740 (93g:47048)
[145] ______, *The centre of spaces of regular operators*, Math. Z. **241** (2002), no. 1, 165–179, DOI 10.1007/s002090200411. MR1930989 (2004b:47069)
[146] ______, *Order and algebra isomorphisms of spaces of regular operators*, Math. Ann. **332** (2005), no. 4, 767–774, DOI 10.1007/s00208-005-0652-4. MR2179776 (2006e:46056)
[147] ______, *Banach lattices with topologically full centre*, Vladikavkaz. Mat. Zh. **11** (2009), no. 2, front matter, 50–60 (English, with English and Russian summaries). MR2529410 (2010h:46024)
[148] Wilbert Wils, *The ideal center of partially ordered vector spaces*, Acta Math. **127** (1971), 41–77. MR0463881 (57 #3819)
[149] Manfred Wolff, *Group actions on Banach lattices and applications to dynamical systems*, Toeplitz centennial (Tel Aviv, 1981), Operator Theory: Adv. Appl., vol. 4, Birkhäuser, Basel, 1982, pp. 501–524. MR669926 (84f:47048)
[150] ______, *Spectral theory of group representations and their nonstandard hull*, Israel J. Math. **48** (1984), no. 2-3, 205–224, DOI 10.1007/BF02761165. MR770702 (86e:46046)
[151] Guido Zapata, *The Stone-Weierstrass theorem and a class of Banach lattice algebras*, Aspects of mathematics and its applications, North-Holland Math. Library, vol. 34, North-Holland, Amsterdam, 1986, pp. 913–942, DOI 10.1016/S0924-6509(09)70299-5, (to appear in print). MR849596 (87j:46045)
[152] Xiao-Dong Zhang, *Some aspects of the spectral theory of positive operators*, Acta Appl. Math. **27** (1992), no. 1-2, 135–142, DOI 10.1007/BF00046644. Positive operators and semigroups on Banach lattices (Curaçao, 1990). MR1184885 (93j:47056)
[153] ______, *On spectral properties of positive operators*, Indag. Math. (N.S.) **4** (1993), no. 1, 111–127, DOI 10.1016/0019-3577(93)90057-6. MR1213328 (94b:47047)
[154] Mirosława Zima, *On the local spectral radius in partially ordered Banach spaces*, Czechoslovak Math. J. **49(124)** (1999), no. 4, 835–841, DOI 10.1023/A:1022413403733. MR1746709 (2001m:47011)

PhD Theses

[155] M. Messerschmidt, *Positive representations on ordered Banach spaces*, Leiden University, advisor M. de Jeu, http://www.math.leidenuniv.nl/scripties/MesserschmidtPhD.pdf, 2013.

[156] K. Muzundu, *Spectral theory in commutatively ordered Banach algebras*, Stellenbosch University, advisor S. Mouton, 2012.

[157] M. Wortel, *Group representations in Banach spaces and Banach lattices*, Leiden University, advisor M. de Jeu, http://www.math.leidenuniv.nl/scripties/WortelPhD.pdf, 2012.

Master's Theses

[158] R. de Jong, *Ordered Banach Algebras*, Leiden University, advisor M. de Jeu, http://www.math.leidenuniv.nl/scripties/DeJongMaster.pdf, 2010.

[159] B. de Rijk, *The Order Bicommuntant*, Leiden University, advisor M. de Jeu, http://www.math.leidenuniv.nl/scripties/DeRijkMaster.pdf, 2012, To be published in a paper.

[160] M. Roelands, *Amenability in Positivity*, Leiden University, advisor M. de Jeu, http://www.math.leidenuniv.nl/scripties/RoelandsMaster.pdf, 2012, To be published in a paper.

[161] J. Rozendaal, *Decomposing positive representations in L^p-spaces for Polish transformation groups*, Leiden University, advisor M. de Jeu, http://www.math.leidenuniv.nl/scripties/RozendaalMaster.pdf, 2011, To be published in a paper.

[162] F. Ruoff, *Positive $C(K)$-representations and positive spectral measures. On their one-to-one correspondence for reflexive Banach lattices*, Leiden University, advisor M. de Jeu, http://www.math.leidenuniv.nl/scripties/MasterRuoff.pdf, 2014, To be published in a paper.

MR1202880 (94d:47027) 47B38 46B42 46E30 47A10 47B60
Abramovich, Y. A. [Abramovich, Yuriĭ Aleksandrovich] (1-INPI);
Arenson, E. L.; **Kitover, A. K.**

★**Banach $C(K)$-modules and operators preserving disjointness. (English summary)**

Pitman Research Notes in Mathematics Series, 277.
Longman Scientific &Technical, Harlow; copublished in the United States with John Wiley &Sons, Inc., New York, 1992. vi+159 *pp. ISBN* 0-582-21020-8

In this research monograph the authors study the properties, in particular the spectral properties, of weighted composition operators in spaces of measurable (vector-valued) functions in the abstract framework of $C(K)$-modules. Here a Banach space X is called a $C(K)$-module if there is also given a closed subalgebra A of the space $L(X)$ of all bounded linear operators which is isometrically isomorphic to an algebra $C(K)$ for some compact Hausdorff space K. A Banach $C(K)$-module X has the property that every closed principal A-submodule has naturally the structure of a Banach lattice. This local structure of Banach lattices allows the authors to extend the notions of order ideal, center, disjointness and disjointness-preserving operators to Banach $C(K)$-modules. The class of Banach $C(K)$-modules includes all Banach lattices, lattice-normed spaces and certain tensor products of Banach lattices with Banach spaces. Many of the results in this monograph are first obtained for Banach lattices and then extended to Banach $C(K)$-modules. Even in the Banach lattice case the authors obtain some remarkable improvements of known results; e.g., they prove the following theorem: If X is a Banach lattice with the property that the Lorentz seminorm $l_X(x)=\inf\{\sup_\alpha\|x_\alpha\|\colon\ 0\le x_\alpha\uparrow|x|\}$ is a norm and if T is a disjointness-preserving operator on X with mutually disjoint powers, then the spectrum $\sigma(T)$ is rotation invariant. This result holds in particular for any Banach function space of measurable functions, and it considerably improves all previously known results in this direction. Another remarkable result is the generalization of Bade's theorem obtained by the authors. They prove that if X is a $C(K)$-module as defined above, then every linear operator (a priori not assumed to be continuous) which maps every $C(K)$-invariant subspace into itself belongs to the weak operator closure of $C(K)$ in $L(X)$. Moreover, this closure is a reflexive operator algebra and can be characterized as the center of X. The authors also obtain a dual version of Bade's theorem: the center of the $C(K)$-module X^* coincides with the space of operators for which each $\sigma(X^*,X)$-closed $C(K)$-invariant subspace of X^* is invariant. For further results, in particular for results about disjointness-preserving operators on Banach $C(K)$-modules, we refer the reader to this interesting book. One warning is however in order: unfortunately (in the reviewer's opinion) the authors came only halfway in adopting common notation and terminology in the English literature on this subject; e.g., they use xdy (which is the common Russian notation) to denote $x\perp y$ and use the term "unextending operator" (a literal translation of the Russian term), which has not been used before in the English literature on this subject, whereas other commonly used terms could have been used. Finally, the reviewer notes that the statement of Proposition 12.15(3) is slightly incorrect. The reader should consult the (correct) proof for the correct statement. *Anton Schep*

MR638681 (83a:46014) 46A40
Aiena, P.; **Oliveri, U.**
Pure states in ordered locally convex Hausdorff spaces.
Rend. Circ. Mat. Palermo (2) **29** (1980), *no. 3,* 427–434 (1981).

Consider a locally convex Hausdorff vector space E ordered by a cone K which has an interior point e. A linear functional f on E is said to be a (K,e)-pure state whenever (a) $f \geq 0$ (i.e., $f(x) \geq 0$ holds for all $x \in K$), (b) $f(e) = 1$, and (c) $0 \leq g \leq f$ implies $g = \lambda f$ for some $0 \leq \lambda \leq 1$. Theorem 1: If M is a subspace of E and $e \in M$, then every (K_M, e)-pure state (where $K_M = K \cap M$) extends to a (K,e)-pure state. Theorem 2: If A is a real commutative ordered Banach algebra and M is a subalgebra with $e \in M$, then each maximal convex algebra ideal of M extends to a maximal convex algebra ideal of A.

C. D. Aliprantis

MR2892579 (Review) 46B42 46H05 47A10 47B65
Alekhno, Egor A. (BE-BELM)
The irreducibility in ordered Banach algebras. (English summary)
Positivity **16** (2012), *no. 1,* 143–176.

The author extends the study of the spectral theory of positive elements in ordered Banach algebras initiated in two papers by H. Raubenheimer and S. Mouton [Indag. Math. (N.S.) **7** (1996), no. 4, 489–502; MR1620116 (99i:46035); Positivity **1** (1997), no. 4, 305–317; MR1660397 (2000a:46070)].

After some preliminaries, the author studies the Frobenius normal form of a positive element, algebras with a disjunctive product, spectral properties of irreducible elements, and some applications to the spectral theory of positive elements. We mention that the theorem about the Frobenius normal form was obtained in the previous literature but from another point of view [J. J. Grobler and C. J. Reinecke, Integral Equations Operator Theory **28** (1997), no. 4, 444–465; MR1465321 (98g:47030); R.-J. Jang and H. D. Victory Jr., Pacific J. Math. **157** (1993), no. 1, 57–85; MR1197045 (93m:47042)].

I think that this work is very elaborate and the results obtained are interesting.

Rodica-Mihaela Dăneţ

References

1. Abramovich, Y.A., Aliprantis, C.D.: An invitation to operator theory. In: Graduate Studies in Mathematics, vol. 50 (2002) MR1921782 (2003h:47072)
2. Abramovich, Y.A., Sirotkin, G.: On order convergence of nets. Positivity **9**(3), 287–292 (2005) MR2188520 (2006f:47048)
3. Aiena, P.: Fredholm and Local Spectral Theory, with Applications to Multipliers. Kluwer, Dordrecht (2004) MR2070395 (2005e:47001)
4. Alekhno, E.A.: Spectral properties of band irreducible operators. In: Proceedings Positivity IV—Theory and Applications, Dresden, Germany, pp. 5–14 (2006) MR2243476 (2007h:47064)
5. Alekhno, E.A.: Some properties of essential spectra of a positive operator. Positivity **11**(3), 375–386 (2007) MR2336203 (2009c:47059)
6. Alekhno, E.A.: Some properties of essential spectra of a positive operator, II. Positivity **13**(1), 3–20 (2009) MR2466225 (2009m:47099)
7. Alekhno, E.A.: The lower Weyl spectrum of a positive operator. Integr. Equ. Oper. Theory **67**(3), 301–326 (2010) MR2660464 (2011f:47070)
8. Aliprantis, C.D., Burkinshaw, O.: Positive Operators. Academic Press, New York (1985) MR0809372 (87h:47086)
9. Aliprantis, C.D., Tourky, R.: Cones and duality. In: Graduate Studies in Mathematics, vol. 84 (2007) MR2317344 (2008k:46012)
10. Burger, I., Grobler, J.J.: Spectral properties of positive elements in Banach lattice algebras. Quaest. Math. **18**(1–3), 261–270 (1995) MR1340482 (96h:46069)
11. Caselles, V.: On the peripheral spectrum of positive operators. Isr. J. Math. **58**(2), 144–160 (1987) MR0901175 (88j:47054)
12. Grobler, J.J.: Spectral theory in Banach lattices. In: Operator Theory: Advances and Applications, vol. 75, pp. 133–172 (1995) MR1322503 (96c:47047)
13. Grobler, J.J., Reinecke, C.J.: On principal T-bands in a Banach lattice. Integr. Equ. Oper. Theory **28**(4), 444–465 (1997) MR1465321 (98g:47030)

14. Jang-Lewis, R.-J., Victory, H.D.: On the ideal structure of positive, eventually compact linear operators on Banach lattices. Pac. J. Math. **157**(1), 57–85 (1993) MR1197045 (93m:47042)
15. Kitover, A.K., Wickstead, A.W.: Operator norm limits of order continuous operators. Positivity **9**(2), 341–355 (2005) MR2188523 (2006i:46028)
16. Krasnosel'skij, M.A., Lifshits, Je.A., Sobolev, A.V.: Positive Linear Systems: The Method of Positive Operators. Heldermann (1989) MR1038527 (91f:47051)
17. Mouton, S., Raubenheimer, H.: More spectral theory in ordered Banach algebras. Positivity **1**(4), 305–317 (1997) MR1660397 (2000a:46070)
18. Raubenheimer, H., Rode, S.: Cones in Banach algebras. Indag. Math. N.S. **7**(4), 489–502 (1996) MR1620116 (99i:46035)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR593826 (81m:47053) 47B55 47B05
Aliprantis, Charalambos D.; **Burkinshaw, Owen**
Positive compact operators on Banach lattices.
Math. Z. **174** (1980), *no. 3,* 289–298.

The authors prove some compactness theorems for positive linear operators with compact majorants. The principal results of interest are as follows. Let E_i $(i=1,2,3,4)$ be Banach lattices and let S_i, T_i $(i=1,2,3)$ be positive linear maps from E_i to E_{i+1} such that $0\le S_i\le T_i$ for $i=1,2,3$. If each T_i is compact, then the composition $S_3S_2S_1$ is likewise compact. Similarly, if either E_1' or E_2 has order continuous norm, then the composition S_2S_1 is compact. These results complement the result of the reviewer and D. H. Fremlin [Israel J. Math. **34** (1979), no. 4, 287–320 (1980); MR0570888 (81g:47037)] that if T_1 is compact, then S_1 is also compact, provided the norms on E_1' and E_2 are order continuous. Some examples are given to show that the results cannot be improved.

Peter Dodds

MR1214920 (94d:47035) 47B65 47A10 47A58
Aràndiga, F. (E-VLNCM); **Caselles, V. [Caselles, Vicent]** (E-VLNCM)
Approximations of positive operators and continuity of the spectral radius.
J. Operator Theory **26** (1991), *no. 1,* 53–71.

One of the main and typical results of this interesting paper is the following one: Let E be a Banach lattice over $\mathbf{C}$ and let (T_n) be a sequence of positive linear operators converging strongly to the operator T. Moreover, assume that $\lim\|(T_n-T)^+\|=0$ and that the spectral radius $r(T)$ of T is a pole of the resolvent with finite–dimensional projection. Then the sequence $(r(T_n))$ of the spectral radii of T_n converges to T.

The paper also contains interesting applications of this and related results, e.g., to a transport equation. Moreover, the paper fascinates by a masterly use of ultrapower techniques which are interesting in themselves.

Unfortunately, there are many irritating misprints of which I mention only the following most misleading ones: (1) p. 56, 6th line from above, read $\bigoplus_{\lambda\in\pi\sigma(T')}$ in place of $\bigoplus_{\lambda\in\sigma(T')}$; (2) same line, read $m_{E'}(\widehat{z})=0$ in place of $m_{E'}(z)\neq 0$; (3) p. 56, 10th line from above, read $\lambda>r(T-PT)$ in place of $\lambda<r(T-PT)$.

{For Part II see the following review.}

Manfred Wolff

MR1156436 (94d:47036) 47B65 47A75
Aràndiga, F. (E-VLNCM-AN); **Caselles, V.** [**Caselles, Vicent**] (F-FRAN)
Approximations of positive operators and continuity of the spectral radius. II.
Math. Z. **209** (1992), *no. 4,* 547–558.

Let E be a weakly sequentially complete Banach lattice and let $T=T_1+T_2$ be a positive linear irreducible operator on E, where $T_1,T_2\geq 0$ and T_2 is an abstract kernel operator. Suppose that the spectral radius $r(T)$ of T is a Riesz point of the spectrum of T. Under these hypotheses the authors prove the following theorem: Let (S_n) be a sequence of positive linear operators converging uniformly on order intervals to T, and assume that $\|(S_n-T)^+\|\to 0$. Let (v_n) be a sequence of positive normalized vectors such that $\lim_n\|S_n v_n - r(S_n)v_n\|=0$. Then $v_n\to v$ where v is the unique positive normalized solution of $Tv=r(T)v$.

This result and some others related to it are proved by an interpolation technique of Davis et al. and by a corresponding convergence result for operators on reflexive Banach lattices of Part I of this paper [J. Operator Theory **26** (1991), no. 1, 53–71; see the preceding review].

Manfred Wolff

MR1297007 (95j:47045) 47B65 47A10 47A58 47B60
Aràndiga, F. (E-VLNC-A); **Caselles, V. [Caselles, Vicent]** (E-BALE-MI)
Approximations of positive operators and continuity of the spectral radius. III. (English summary)
J. Austral. Math. Soc. Ser. A **57** (1994), *no. 3,* 330–340.

In Parts I and II [J. Operator Theory **26** (1991), no. 1, 53–71; MR1214920 (94d:47035); Math. Z. **209** (1992), no. 4, 547–558; MR1156436 (94d:47036)] the authors proved among other things the following interesting theorems: Let $T_n, T \geq 0$ be bounded linear operators on a weakly sequentially complete Banach lattice E such that (T_n) converges uniformly on order intervals to T and such that $(\|(T_n - T)^+\|)$ converges to 0. Under certain additional hypotheses on T the sequence $(r(T_n))$ of the spectral radii converges to the spectral radius $r(T)$ of T. In the paper under review they generalize these results including estimates on the speed of convergence. Let us mention in particular the following two interesting results. Theorem 2.2: Let E be an arbitrary Banach lattice, let T, T_n be as above, and assume that (T_n) converges strongly to T and that $(\|(T_n - T)^+\|)$ converges to 0. If $r(T)$ is a Riesz point of the spectrum $\sigma(T)$ then $r(T_n)$ is a Riesz point of $\sigma(T_n)$ for n sufficiently large.

Theorem 3.2: Let E be reflexive and assume that $T \geq 0$ is irreducible and satisfies some additional hypotheses. Assume that $r(T)$ is a Riesz point. Then for any λ of the peripheral spectrum of T there exists a constant $k > 0$ and a sequence (λ_n) with $\lambda_n \in \sigma(T_n)$ such that $|\lambda_n - \lambda| \leq k\|T_n x - Tx\|$ where x is the unique (up to a complex number of modulus 1) normalized solution of $Tx = \lambda x$.

The main method in the proofs is a sophisticated use of ultrapowers. It is surprising that this method yields such strong estimates. In my opinion these methods allow generalizations of the results above to the case of discrete approximation as considered in [H. O. Fattorini, *The Cauchy problem*, Addison-Wesley, Reading, MA, 1983; MR0692768 (84g:34003)(Chapter 5.7)]. These generalizations would have applications, e.g., to numerical problems.

Manfred Wolff

MR631633 (83h:47027) 47B55 43A22
Arendt, Wolfgang
On the o-spectrum of regular operators and the spectrum of measures.
Math. Z. **178** (1981), *no. 2,* 271–287.

If E is a Banach lattice, the linear span of the positive operators on E is denoted $L^r(E)$ and is called the space of regular operators on E. There is a norm $\|\cdot\|_r$ on $L^r(E)$ which dominates the operator norm and is defined by the formula $\|T\|_r = \inf\{\|S\| : S \geq 0, |Tz| \leq S|z| \text{ for all } z \in E\}$. With respect to this norm $L^r(E)$ is a Banach algebra, and the spectrum of a regular operator in this algebra is called the o-spectrum of the operator. Generally, the o-spectrum of a regular operator is larger than the usual spectrum of the operator. The main objective of the first part of the paper under review is to show that if T in $L^r(E)$ is r-compact in the sense that T is the r-norm limit of finite rank regular operators, then the spectrum of T and o-spectrum of T coincide. The author gives an example of a compact positive operator that is not r-compact and whose o-spectrum is uncountable. The second part of the paper is an application of the first part to the spectral analysis of convolution operators.

Paul S. Muhly

MR908795 (89g:47048) 47B55 46B30 47B05 47D15 47D30

Arendt, Wolfgang (D-TBNG); **Schwarz, Hans-Ulrich** (DDR-KMU)

Ideale regulärer Operatoren und Kompaktheit positiver Operatoren zwischen Banachverbänden. (German) [Ideals of regular operators and compactness of positive operators between Banach lattices]

Math. Nachr. **131** (1987), 7–18.

The subspace $K(E,F)$ of all compact operators in the space of all bounded linear operators $L(E,F)$ between two Banach spaces E and F is, for the sequence spaces l^p $(1 \le p < \infty)$ and c_0, of a special nature. It is well known that if $E = F = c_0$ or $E = F = l_p$ $(1 \le p < \infty)$, then $K(E) := K(E,E)$ is the only nontrivial two-sided ideal of the algebra $L(E) := L(E,E)$. If $1 \le q < p < \infty$, then $K(l_p, l_q) = L(l_p, l_q)$. The purpose of this paper is to investigate the same question for the space of compact and order-bounded linear operators between Banach lattices. To get the flavor of the many interesting results the paper contains we quote the following: (1) For all $1 < p < \infty$, the only two-sided ideal of $L^r(l_p)$, the Banach algebra of all order-bounded linear operators on l_p, which is also a sublattice of $L^r(l_p)$, is the ideal $K^r(l_p)$ of all the compact order-bounded linear operators. (2) If $1 \le q < p < \infty$, then $L^r(l_p, l_q) = K^r(l_p, l_q)$ and $L^r(c_0, l_q) = K^r(c_0, l_q)$. (3) The dual space of a Banach lattice E has an order-continuous norm if and only if every positive operator of E into l_1 is weakly compact and in that case $L^r(E, l_1) = K^r(E, l_1)$.

W. A. J. Luxemburg

MR691284 (84c:47048) 47D30 47B55
Arendt, W.; **Sourour, A. R.**
Ideals of regular operators on l^2.
Proc. Amer. Math. Soc. **88** (1983), *no. 1,* 93–96.

The paper provides an order-theoretic analogue of the fact that in the Banach algebra of all bounded operators on a separable Hilbert space the set of all compact operators is the only nontrivial closed ideal. A bounded operator A in l_2 with matrix (a_{mn}) is said to be regular whenever the matrix $(|a_{mn}|)$ also defines a bounded operator $|A|$ (called the modulus of A). Every finite rank operator is regular. The vector space $\mathcal{L}^r$ of all regular operators is a Banach subalgebra of $\mathcal{L}(l_2)$, and under the r-norm $\|A\|_r := \||A|\|$ is also a Banach lattice. An algebra-ideal $\mathcal{I}$ of $\mathcal{L}^r$ which is also a lattice-ideal (i.e., $|B| \leq |A|$ and $A \in \mathcal{I}$ imply $B \in \mathcal{I}$) is referred to as an ideal of $\mathcal{L}^r$. The norm closure in $\mathcal{L}^r$ of the finite rank operators is denoted by $\mathcal{K}^r$. The main results: (1) $\mathcal{K}^r$ is the only nontrivial closed ideal in $\mathcal{L}^r$; and (2) $\mathcal{K}^r$ contains every proper ideal of $\mathcal{L}^r$.

When l_2 is replaced by $L_2 = L_2[0,1]$ the situation is different. In this case, $\mathcal{K}^r(L_2)$ is the smallest closed ideal in $\mathcal{L}^r(L_2)$. The paper concludes with the following interesting question: Is $\mathcal{K}^r(L_2)$ the only nontrivial closed ideal in the Banach lattice algebra of kernel operators on L_2? *C. D. Aliprantis*

References

1. C. D. Aliprantis and O. Burkinshaw, *Positive compact operators on Banach lattices* Math. Z. **174** (1980), 289-298. MR0593826 (81m:47053)
2. W. Arendt, *On the o-spectrum of regular operators and the spectrum of measures*, Math. Z. **179** (1981), 271-287. MR0631633 (83h:47027)
3. P. Dodds and D. H. Fremlin, *Compact operators in Banach lattices*, Israel J. Math. **34** (1979), 287-320. MR0570888 (81g:47037)
4. H. H. Schaefer, *Banach lattices and positive operators*, Springer-Verlag, Berlin-Heidelberg-New York, 1974. MR0423039 (54 #11023)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR849713 (87j:47052) 47B55 47A10 47A55
Arendt, W. (D-TBNG); **Sourour, A. R.** (3-VCTR)
Perturbation of regular operators and the order essential spectrum.
Nederl. Akad. Wetensch. Indag. Math. **48** (1986), *no. 2,* 109–122.

For regular operators on a Banach lattice the authors introduce the order essential spectrum and the order Weyl spectrum. Their aim is to investigate the relations between these spectra and the notions of spectrum, essential spectrum, Weyl spectrum and order spectrum. Moreover, they consider perturbation-theoretical problems with respect to these spectra. *M. Demuth*

MR1020394 (91f:46072) 46J10 06F25 46H10
Basly, Mohamed (TN-TUNIS)
Produit naturel pour les FF-algèbres de Banach réticulées de type $C(X)$. (French. English summary) [Natural product for lattice-ordered Banach FF-algebras of type $C(X)$]
Funct. Approx. Comment. Math. **18** (1989), 63–66.

An FF-algebra is a (real) lattice-ordered Banach algebra $A(+,*)$ such that $x\wedge y=0$ implies $x*y=0$. For X a compact space, $C(X)$ denotes the (Riesz) space of (real-valued) continuous functions on X with the usual sup norm. The author seems to say that $(C(X),+,*)$ is an FF-algebra if and only if there is an $\alpha\in C(X)$ such that $0\le\alpha\le 1$ and $\alpha(t)=1$ for some $t\in X$ for which $f*g$ is the pointwise product of f, g, and α (and hence is commutative). (This latter statement is inferred by the reviewer from a more ponderous statement in the language of abstract measures.) It follows that, for any $x\in X$, $\{f\in C(X,+,*)\colon f(x)=0\}$ is a maximal ideal of $C(X,+,*)$ if and only if $x\in\{t\in X\colon\ \alpha(t)>0\}$. The paper depends heavily on a familiarity with the contents of a paper by E. Scheffold [Math. Z. **177** (1981), no. 2, 193–205; MR0612873 (82f:46060)]. The reviewer is unable to follow the details of the proof of the main result.

REVISED (May, 2003)

Current version of review. Go to earlier version. *Melvin Henriksen*

MR2379731 (2009b:46102) 46H05 46B40 46H10 47B60
Behrendt, D. [Behrendt, Darren Robin] (SA-UJ);
Raubenheimer, H. [Raubenheimer, Heinrich] (SA-UJ)
On domination of inessential elements in ordered Banach algebras. (English summary)
Illinois J. Math. **51** (2007), *no. 3,* 927–936.

The authors consider questions of the following type: Let A be a Banach algebra (over $\mathbb{C}$, with an identity, and semiprime) ordered by a so-called algebra cone, and let a and b be elements of A such that $0 \leq a \leq b$, and suppose that b has a certain property. Does a then inherit that property? *Volker Runde*

References

1. C. D. Aliprantis and O. Burkinshaw, *Positive compact operators on Banach lattices*, Math. Z. **174** (1980), 289–298. MR 593826 (81m:47053) MR0593826 (81m:47053)
2. W. Arendt, *On the o-spectrum of regular operators and the spectrum of measures*, Math. Z. **178** (1981), 271–287. MR 631633 (83h:47027) MR0631633 (83h:47027)
3. B. Aupetit, *A primer on spectral theory*, Universitext, Springer-Verlag, New York, 1991. MR 1083349 (92c:46001) MR1083349 (92c:46001)
4. R. M. Brits, L. Lindeboom, and H. Raubenheimer, *On the structure of rank one elements in Banach algebras*, Extracta Math. **18** (2003), 297–309. MR 2056597 (2006a:46058) MR2056597 (2006a:46058)
5. J. Martínez and J. M. Mazón, *Quasi-compactness of dominated positive operators and C_0-semigroups*, Math. Z. **207** (1991), 109–120. MR 1106817 (92g:47048) MR1106817 (92g:47048)
6. S. Giotopoulos and M. Roumeliotis, *Algebraic ideals of semiprime Banach algebras*, Glasgow Math. J. **33** (1991), 359–363. MR 1127528 (92g:46060) MR1127528 (92g:46060)
7. R. Harte, *On rank one elements*, Studia Math. **117** (1995), 73–77. MR 1367694 (96i:46055) MR1367694 (96i:46055)
8. H. d. T. Mouton and S. Mouton, *Domination properties in ordered Banach algebras*, Studia Math. **149** (2002), 63–73. MR 1881716 (2003e:46076) MR1881716 (2003e:46076)
9. S. Mouton and H. Raubenheimer, *More spectral theory in ordered Banach algebras*, Positivity **1** (1997), 305–317. MR 1660397 (2000a:46070) MR1660397 (2000a:46070)
10. J. Puhl, *The trace of finite and nuclear elements in Banach algebras*, Czechoslovak Math. J. **28(103)** (1978), 656–676. MR 506439 (81a:47024) MR0506439 (81a:47024)
11. H. Raubenheimer, *r-asymptotically quasifinite rank operators and the spectrum of measures*, Quaestiones Math. **10** (1986), 97–111. MR 857772 (87m:47090) MR0857772 (87m:47090)
12. H. Raubenheimer and S. Rode, *Cones in Banach algebras*, Indag. Math. (N.S.) **7** (1996), 489–502. MR 1620116 (99i:46035) MR1620116 (99i:46035)
13. H. Raubenheimer and T. J. D. Wilkins, *On a spectral condition in Banach algebras*, Bull. London Math. Soc. **28** (1996), 305–310. MR 1374410 (96k:46084) MR1374410 (96k:46084)

MR1083344 (92d:46054) 46B42 46J99 47B60 47B65
Bernau, S. J. (1-TXEP); **Huijsmans, C. B.** (NL-LEID)
On the positivity of the unit element in a normed lattice ordered algebra.
Studia Math. **97** (1990), *no. 2,* 143–149.

Let $T: E \to E$ be a linear operator on a normed Riesz space with T greater than or equal to the identity operator. An elementary proof is provided for the result that if T is Cesàro bounded (i.e., the sequence $(\|\sum_{k=0}^{n} T^k\|/(n+1))$ is bounded), equivalently Abel bounded, then T is equal to the identity operator. The condition of power bounded (i.e. $\sup \|T^n\| < \infty$) or a contraction implies Cesàro bounded. It is also shown by the same arguments that if A is a lattice normed algebra with unit e and $\|e\| \leq 1$, then $e \geq 0$. This was first shown by Scheffold in the case of complete norm. *W. A. Feldman*

MR2466228 (2010a:46107) 46H05 46B99 47A10
Braatvedt, Gareth (SA-UJ); **Brits, Rudi** (SA-UJ);
Raubenheimer, Heinrich (SA-UJ)
Gelfand-Hille type theorems in ordered Banach algebras. (English summary)
Positivity **13** (2009), *no. 1,* 39–50.

Let $\mathcal{A}$ be a Banach algebra. A subset $C \subset \mathcal{A}$ is said to be a cone if $C + C \subset C$, $\lambda C \subset C$ for each $\lambda \geq 0$, $C \cdot C \subset C$ and the identity $I \in \mathbb{C}$. A cone C in a Banach algebra C induces an order in $\mathcal{A}$, namely, for a and b in $\mathcal{A}$ we say that $a < b$ if $b - a$ is in C. An element a in $\mathcal{A}$ is said to be Cesàro bounded if there is an $M > 0$ such that

$$\left\| \sum_{k=0}^{n} a^k \right\| \leq M(n+1) \text{ for } n = 0, 1, 2 \ldots ,$$

and is said to be Abel bounded if there is an $M > 0$ such that

$$\left\| (1-\theta) \sum_{k=0}^{\infty} \theta^k a^k \right\| \leq M \quad \text{ for each } 0 < \theta < 1.$$

J. J. Grobler and C. B. Huijsmans [Quaestiones Math. **18** (1995), no. 4, 397–406; MR1354120 (96i:47001)] and J. Sánchez-Álvarez, J. Zemánek and the reviewer [Proc. London Math. Soc. (3) **91** (2005), no. 3, 761–788; MR2180462 (2006j:47053)] showed that uniformly Abel bounded, which means

$$\left\| (1-\theta) \sum_{k=0}^{n} \theta^k a^k \right\| \leq M \quad \text{ for each } 0 < \theta < 1 \text{ and } n = 0, 1, 2, \ldots$$

is equivalent to Cesàro bounded. Thus both of them imply Abel boundedness.

On the other hand, there are several extensions of a theorem of I. Gelfand [Rec. Math. [Mat. Sbornik] N. S. **9 (51)** (1941), 49–50; MR0004635 (3,36d)], which asserts that if the spectrum $\sigma(x) = \{1\}$ and x and x^{-1} are Abel bounded, then $x = I$.

The authors of the present paper ask for conditions on C under which Abel boundedness implies Cesàro boundedness. They prove the following assertion:

Assume that C is closed (in the algebra) and normal, that is, there is $\alpha > 0$ such that if $0 < x < y$, then $\|x\| \leq \alpha \|y\|$. Then Abel boundedness implies Cesàro boundedness.

As a corollary, they show that if any element x with single spectrum $\sigma(x) = \{1\}$ and x and x^{-1} are both Abel bounded, then x is the identity. The authors also show other interesting results in which they relate several kinds of boundedness of the powers. They also provide conditions for the nilpotency of an element $x - I$.

Alfonso Montes-Rodríguez

References

1. G.R. Allan, *Power-bounded elements and radical Banach algebras*, Linear operators, Banach Center Publications, **38** (1997), 9–16. MR1456997 (98c:46094)
2. G.R. Allan, T.J. Ransford, *Power-dominated elements in a Banach algebra*, Stud. Math., **94** (1989), 63–79. MR1008239 (91a:46050)
3. D. Drissi, J. Zemánek, *Gelfand-Hille theorems for Cesàro means*, Quaestiones Math., **23** (2000), 375–381. MR1809944 (2001m:47019)
4. I. Gelfand, *Zur Theorie der Charaktere der abelschen topologischen Gruppen*, Rec.

Math. N.S. (Math. Sb.) **9** (51) (1941), 49–50. MR0004635 (3,36d)

5. J.J. Grobler, C.B. Huijsmans, *Doubly Abel bounded operators with single spectrum*, Quaestiones Math., **18** (1995), 397–406. MR1354120 (96i:47001)
6. E. Hille, *On the theory of characters of groups and semi-groups in normed vector rings*, Proc. Nat. Acad. Sci. USA, **30** (1944), 58–60. MR0009702 (5,189a)
7. M. Mbekhta, J. Zemánek, *Sur le théorème ergodique uniforme et le spectre*, C.R. Acad. Sci. Paris, **317** (1993), 1155–1158. MR1257230 (95b:47010)
8. A. Montes Rodríguez, J. Sánchez-Álvarez, J. Zemánek, *Uniform Abel-Kreiss boundedness and the extremal behaviour of the Volterra operator*, Proc. Lond. Math. Soc., (3) **91** (2005), 761–788. MR2180462 (2006j:47053)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR0234285 (38 #2602) 46.55
Brown, G. [Brown, Gavin]
Type 0 semi-algebras in Banach algebras.
J. London Math. Soc. **43** 1968 482–486

A semi-algebra in a Banach algebra B is a subset A of B such that $x+y$, αx and $x \cdot y$ belong to A whenever x and y are in A, and α is real and ≥ 0. It is said to be closed if it is closed in the norm topology of B. If A is closed the difference algebra $A-A$ is a Banach algebra in the so-called cone norm.

In earlier work on semi-algebras in the Banach algebra $C(X)$ of real continuous functions on a compact space X, Bonsall defined a type n semi-algebra to be one such that $f \in A$ implies $f^n/(1+f) \in A$, where n is a fixed non-negative integer. (See F. F. Bonsall [Proc. London Math. Soc. (3) **10** (1960), 122–140; MR0112034 (22 #2892); Proc. Internat. Sympos. Linear Spaces (Jerusalem, 1960), pp. 101–114, Jerusalem Academic Press, Jerusalem, 1961; MR0154105 (27 #4064)], and E. J. Barbeau [Trans. Amer. Math. Soc. **120** (1965), 1–16; MR0182889 (32 #371)].)

This paper, presenting work from the author's Ph.D. thesis ["Norm and stability properties of semi-algebras", Univ. Newcastle upon Tyne, Newcastle upon Tyne, 1966], generalizes results on type 0 semi-algebras to the case where B is any commutative Banach algebra, with carrier space M_B and principal semi-algebra B^+ (defined as $\{x \in B\colon m(x) \geq 0$ all $m \in M_B\}$). Let A be a type 0 semi-algebra, that is, $x \in A$ implies $(1+x)^{-1} \in A$, and let A be closed. The main results are: (1) $S = A-A$ is closed in B. Also, S is strictly real ($m(x)$ is real for all $x \in S$, $m \in M_S$) and A is its principal semi-algebra. (2) If the norm and the spectral radius coincide on A, then there is a compact Hausdorff space X such that A is isometrically isomorphic to $C^+(X)$.

{There is an error: The inequality (2) on p. 485 is false, but it is not hard to find another proof that the z_ε converge. Misprints: p. 482, line 5: for the second B read A. p. 485, line 8*: for X read S.} *J. D. Pryce*

MR0244767 (39 #6081) 46.55
Brown, Gavin
Norm properties of a class of semi-algebras.
J. London Math. Soc. **44** 1969 329–339

The author exhibits a class of Banach algebras in which a natural positive semi-algebra can be identified by a simple multiplicative property of the norm. From the author's introduction: "Let $C[0,1]$ denote the Banach algebra of all real-valued continuous functions on the closed unit interval under the uniform norm. For any positive integer n, let A_n denote the subset of $C[0,1]$ consisting of those non-negative functions whose first n differences are non-negative. The closed semi-algebras $\{A_n\}_{n\in N}$ have already motivated a large part of semi-algebra theory. Here we study the difference algebras of these semi-algebras and their cone norms. Our starting point is the following result proved by F. F. Bonsall [*Lectures on some fixed point theorems of functional analysis*, Tata Inst. Fund. Res., Bombay, 1962; MR0198173 (33 #6332)]: If A is a closed semi-algebra in a (real) Banach algebra, then the difference algebra $S=A-A$ is itself a Banach algebra under the cone norm and $\|\ \|_S$. $\|\ \|_S$ is the Minkowski functional of the absolutely convex hull of the unit ball of A."

The main result shows that for each n, A_n can be recovered from its difference algebra S_n by the formula $\|f^{n+1}\|_n=(\|f\|_n)^{n+1}$ if and only if $f\in\pm A_n$, where $\|\ \|_n$ denotes the cone norm on S_n. In the case $n=1$ this appears in the form "a function of bounded variation is monotonic and of constant sign if and only if the variation of its square equals the square of its variation". The proof, by induction, involves ingenious computation.

An integral representation for $\|f\|_n$ is obtained. It is shown that S_n contains inverses of its non-vanishing functions, and hence that its carrier space is $[0,1]$; however, that of the algebra $S_\infty=\bigcap S_n$ is the unit disc. Finally, it is shown that S_n is an abstract (L) space with respect to the cone norm and the order induced by A_n.

Like the author's earlier work [J. London Math. Soc. **43** (1968), 482–486; MR0234285 (38 #2602)], this article forms part of his Ph.D. thesis ["Norm and stability properties of semi-algebras", Univ. of Newcastle, Newcastle upon Tyne, 1966]. See also F. F. Bonsall [Proc. London Math. Soc. (3) **10** (1960), 122–140; MR0112034 (22 #2892)].

J. D. Pryce

MR1340482 (96h:46069) 46H05 06F25 46B42 47B60 47B65
Burger, I. (SA-POTCH); **Grobler, J. J. [Grobler, Jacobus J.]** (SA-POTCH)
Spectral properties of positive elements in Banach lattice algebras. (English summary)
First International Conference in Abstract Algebra (Kruger Park, 1993).
Quaestiones Math. **18** (1995), *no. 1-3,* 261–270.

The authors study Banach lattice algebras with a multiplicative unit, looking for spectral properties of positive elements which are analogous to those of positive operators on Banach lattices. The method used is to take a Dedekind-complete Banach lattice algebra E with a separating algebraic unit element and right continuous multiplication and to postulate the existence of a large enough set of one-dimensional elements, at least one of which must be order continuous and one of which must be idempotent. For compact $x \in E$ the corresponding operator ${}_xT$ of multiplication on the left by x is compact. Suppose $x > 0$ and x is order continuous and irreducible. Let $r(\cdot)$ denote spectral radius. The authors prove that if x is also compact then $r(x) = r({}_xT) > 0$. Additionally, they show (Perron-Jentzsch) that if x^k is compact then $r(x) > 0$ and $r(x)$ is an eigenvalue with algebraic multiplicity 1 whose eigenvector u satisfies $eu > 0$ for all nonzero components e of the unit element. For further details and a Frobenius-type theorem giving cyclic behavior of the eigenvalues of modulus $r(x)$ we refer to the paper itself. The paper draws on earlier work of the second author [Nederl. Akad. Wetensch. Indag. Math. **49** (1987), no. 4, 381–391; MR0922442 (88k:47053)].

{For the entire collection see MR1340467 (96c:00020)}

S. J. Bernau

MR1082002 (91k:47083) 47B65
Burlando, Laura (I-GENO)
Monotonicity of spectral radius for positive operators on ordered Banach spaces.
Arch. Math. (Basel) **56** (1991), *no. 1,* 49–57.

Let $T \in B(E)$, where E is a Banach space partially ordered by a cone K and where $B(E)$ denotes the Banach space of bounded linear operators mapping E into E. Let $r(T)$ denote the spectral radius of T. It is well known that if K is generating and normal, then the following very natural implication holds: $0 \le S \le T$, $S, T \in B(E)$, implies that $r(S) \le r(T)$. The situation changes dramatically if the cone K fails to possess some of the above properties. The author shows that there exist a Banach space partially ordered by a cone K which is normal and total but not generating, and a pair of K-nonnegative bounded linear operators S and T such that $0 \le S \le T$, whilst $r(S) > r(T)$. Another example shows that the monotonicity of the spectral radius may fail to hold if E is ordered by a cone K which is generating but not normal. As one can guess, the above examples are related by the famous characterization of generating cones via their duals due to Kreĭn and Grossberg, and vice versa.

Other types of negative results are associated with the implication that $0 \le S \le T$, T compact, implies S^3 is compact when the order cone K is a Banach lattice cone. The author constructs a total cone K such that E is not a Banach lattice, and operators S and T such that $0 \le S \le T$, where S^3 may not be compact and, moreover, S may not have any compact power. *I. K. Marek*

MR834316 (87i:47050) 47B55 46B30
Caselles, V. [Caselles, Vicent]
On irreducible operators on Banach lattices.
Nederl. Akad. Wetensch. Indag. Math. **48** (1986), *no. 1,* 11–16.

Let E be a (complex) Dedekind complete Banach lattice. The space of order continuous linear functionals on E is denoted by $E_{\rm n}^*$, and we assume that $E_{\rm n}^*$ separates the points of E. Denote by $(E_{\rm n}^* \otimes E)^{\rm dd}$ the band generated by the order continuous finite rank operators on E in the space $\mathcal{L}_{\rm b}(E)$ of all order bounded operators on E. Note that, if E is a Banach function space, then $(E_{\rm n}^* \otimes E)^{\rm dd}$ is precisely the set of all absolute integral operators in E. A positive operator T on E is called a Harris operator if there exists a natural number h and an operator $K \in (E_{\rm n}^* \otimes E)^{\rm dd}$ such that $0 < K \leq T^h$. Furthermore, a positive operator T in E is said to be (band) irreducible if $\{0\}$ and E are the only T-invariant bands. By the well-known Ando-Krieger theorem, if $0 < T \in (E_{\rm n}^* \otimes E)^{\rm dd}$ is irreducible, then the spectral radius $r(T) > 0$ [see, e.g., A. C. Zaanen, *Riesz spaces, II*, North-Holland, Amsterdam, 1983; MR0704021 (86b:46001)].

The main purpose of the paper under review is to extend the Ando-Krieger theorem to Harris operators. In fact, it is shown that if T is a positive order continuous irreducible Harris operator in E, then $r(T) > 0$. In addition, using this result, some other conditions on a positive operator T which imply that $r(T) > 0$ are discussed. *B. de Pagter*

MR901175 (88j:47054) 47B55 47A10
Caselles, Vicent (E-VLNC)
On the peripheral spectrum of positive operators.
Israel J. Math. **58** (1987), *no. 2,* 144–160.

From the introduction: "An interesting problem in the theory of positive operators in Banach lattices is to know what properties of $T \in \mathcal{L}(E)$, where E is a Banach lattice, are inherited by $S \in \mathcal{L}(E)$ if we know that $0 \leq S \leq T$."

The main result of this paper is of the type described above, namely: Let E be a Banach lattice and let $S, T \in \mathcal{L}(E)$ be such that $0 \leq S \leq T$ and their spectral radii are equal, $r(T) = r(S)$. Then, if $r(T)$ is a Riesz point of $\sigma(T)$ (that is, if $r(T)$ is a pole of the resolvent $(z - T)^{-1}$ whose residue is of finite rank), then $r(S)$ is a Riesz point of $\sigma(S)$.

Irina Popa

MR1112090 (92h:47006) 47A10 47B07 47B60
Cherdak, V. B.
On the order spectrum of r-compact operators in lattice-normed spaces. (Russian)
Sibirsk. Mat. Zh. **32** (1991), *no. 1,* 148–152, 221; *translation in Siberian Math. J.* **32** (1991), *no. 1,* 123–126.

Let T be a regular operator on a Dedekind complete Banach lattice X. Recall that apart from the traditional spectrum $\sigma(T)$ there exists an o-spectrum $\sigma_o(T)$ provided we consider T as an element of the Banach algebra of all regular operators. W. Arendt [Math. Z. **178** (1981), no. 2, 217–287; MR0631633 (83h:47027)] found the conditions ensuring that $\sigma(T) = \sigma_o(T)$. In the present article Arendt's method is generalized to prove a similar result for r-compact operators on lattice-normed spaces.

Yu. A. Abramovich

MR0458153 (56 #16356) 46E30 46J99
Chiţescu, Ion
Köthe spaces that are Banach algebras with unit.
Bull. Math. Soc. Sci. Math. R. S. Roumanie (N.S.) **18(66)** (1974), *no. 3-4,* 269–271 (1976).

A function space is the collection of all real- or complex-valued functions which are measurable on a measurable space (X, ε, μ); such a space is denoted by M. M_+ denote the set of all positive extended real-valued functions on X. A function norm is a positive extended real-valued function on M_+ which respects the natural order in M_+ and which is positive homogeneous and subadditive on M_+. It is shown in this paper that the only function norm ρ, up to equivalence as a Banach space, that makes L_ρ a Banach algebra with a unit is the L_∞ norm. Here if ρ denotes a function norm then L_ρ denotes that subspace V of functions f in M such that $\rho(|f|) < \infty$.

REVISED (1979)

Current version of review. Go to earlier version. *M. Rajagopalan*

MR1006639 (90g:47055) 47B38 28B15 46G10 46J99 47A67
Choy, Stephen T. L. (SGP-SING)
Positive operators and algebras of dominated measures.
Rev. Roumaine Math. Pures Appl. **34** (1989), *no. 3,* 213–219.

Let S be a locally compact semitopological semigroup and $\mathcal{B}$ the Borel sets of S. Let A, B be Banach algebras and let B be ordered by the closed cone K. If $C_0(S,A)$ is the space of all A-valued continuous functions on S which vanish at ∞ equipped with the sup-norm, then a continuous linear operator $T\colon C_0(S,A) \to B$ has a measure $m\colon \mathcal{B} \to L(A,B'')$, the space of continuous linear operators from A into B'', which represents T. If A is a B^* algebra, then T is positive if and only if $T(ff^*) \in K$ for $f \in C_0(S,A)$. The author shows that T is positive if and only if $m(E)(xx^*) \in K^{**}$ (dual cones) for every $E \in \mathcal{B}$, $x \in A$.

A weakly compact operator $T\colon C_0(S) \to A$ is said to be dominated if its representing measure has bounded variation. Denote all such operators by $D(S,A)$. The author gives a description of $D(S,A)$ in terms L^1 subalgebras with respect to positive measures and then uses the characterization to describe the dual of $D(S,A)$. *C. W. Swartz*

MR0491402 (58 #10658) 06A70 47B55
Dai, Taen Yu; **DeMarr, Ralph**
Positive derivations on partially ordered linear algebra with an order unit.
Proc. Amer. Math. Soc. **72** (1978), *no. 1,* 21–26.

Let A be a real linear associative algebra which is partially ordered so that it becomes a directed partially ordered linear space and $0 \le xy$ whenever $0 \le x, y \in A$. Suppose also A is Dedekind σ-complete ($x_n \in A$, $x_1 \ge x_2 \ge \cdots \ge 0$ implies $\inf\{x_n\}$ exists) and has an order unit u (for each $x \in A$ there exists a real number δ such that $-\delta u \le x \le \delta u$). A positive derivation f on A is a linear map from A into itself such that $f(xy) = xf(y) + f(x)y$ for all x, y in A with the additional property that $f(x) \ge 0$ whenever $x \ge 0$. An element x in A is a generalized nilpotent if for each real number $b > 0$, there exists v in A (depending on b) satisfying the inequality $-v \le b^n x^n \le v$ for all positive integers n. Note that every nilpotent is a generalized nilpotent. The authors' main result is that the range of any positive derivation f on A is a set of generalized nilpotents. The motivation for this theorem is the second author's result that every Banach algebra $B(X)$ of bounded linear operators on a Banach space X can be ordered so that it has all the properties of the algebra A given above [Canad. J. Math. **19** (1967), 636–643; MR0212579 (35 #3450)]. The authors also give several examples which show that (i) the existence of an order unit in A is necessary and (ii) the range may include generalized nilpotents which are not nilpotent. Note that if A is also a Banach algebra where $x \le y$ implies $\|x\| \le \|y\|$, the theorem implies that the range of f is a set of quasinilpotents. *Nicholas P. Jewell*

References

1. T. Y. Dai, *On some special classes of partially ordered linear algebras*, J. Math. Anal. Appl. **40** (1972), 649-682. MR0316342 (47 #4890)
2. T. Y. Dai and R. DeMarr, *Partially ordered linear algebras with multiplicative diagonal projection map*, Trans. Amer. Math. Soc. **224** (1976), 179-187. MR0419330 (54 #7352)
3. R. DeMarr, *On partially ordering operator algebras*, Canad. J. Math. **19** (1967), 636-643. MR0212579 (35 #3450)
4. R. V. Kadison and J. R. Ringrose, *Derivations and automorphisms of operator algebras*, Comm. Math. Phys. **4** (1967), 32-63. MR0206735 (34 #6552)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR3125128 (Review) 47L10 46H10
Dales, H. G. [Dales, H. Garth] (4-LANCF-MS); **Kania, Tomasz** (4-LANCF-MS); **Kochanek, Tomasz** (PL-SILS-IM); **Koszmider, Piotr** (PL-PAN); **Laustsen, Niels Jakob** (4-LANCF-MS)

Maximal left ideals of the Banach algebra of bounded operators on a Banach space. (English summary)

Studia Math. **218** (2013), *no. 3,* 245–286.

Consider a Banach space E and let $\mathcal{B}(E)$ be the Banach algebra of bounded linear operators on E. Given a subset Γ of $\mathcal{B}(E)$, let $\mathcal{L}_\Gamma$ be the smallest (not necessarily closed) left ideal of $\mathcal{B}(E)$ which contains Γ. Motivated by previous work of the first author and W. Żelazko [Studia Math. **212** (2012), no. 2, 173–193; MR3008440], this paper asks two questions: Firstly, does $\mathcal{B}(E)$ always contain a maximal left ideal not equal to $\mathcal{L}_\Gamma$ for a finite Γ (that is, not finitely-generated)? Secondly, is every finitely-generated maximal left ideal fixed, that is, of the form $\{T : Tx = 0\}$ for a fixed vector x?

The paper proves a number of key results. Theorem 1.1 shows that for any maximal left ideal L, either L is fixed or it contains all finite-rank operators; notice that these properties are mutually exclusive, so this is a genuine dichotomy. Theorem 1.3 is another dichotomy result: if L is a closed left ideal, and either L is finitely-generated or E is reflexive, then either L is contained in a fixed (maximal) left ideal or L contains the finite-rank operators.

Theorem 1.4 shows that the second question has a positive answer for many classical, and exotic, Banach spaces such as: E has a Schauder basis and is complemented in E^{**}, $E = c_0$, E is a Hilbert space, E is a Banach space which has a few operators. However, the authors construct a counter-example in general: if X is the Argyros-Haydon space [S. A. Argyros and R. G. Haydon, Acta Math. **206** (2011), no. 1, 1–54; MR2784662 (2012e:46031)], then with $E = X \oplus \ell_\infty$, the authors give an explicit description of a maximal two-sided ideal in $\mathcal{B}(E)$ of codimension one which is singly generated, but not fixed. However, in Theorem 1.6 it is shown that E does not give a negative answer to the first question.

In addition to building the necessary theory to prove these results, Section 3 of the paper studies the question of counting the number of maximal left ideals in $\mathcal{B}(E)$. The paper is very well written and carefully referenced, and the reader can learn a lot of interesting facts about operators on Banach spaces from this paper. Some open questions are provided at the end. *Matthew D. Daws*

References

1. F. Albiac and N. J. Kalton, *Topics in Banach Space Theory,* Grad. Texts in Math. 233, Springer, New York, 2006. MR2192298 (2006h:46005)
2. D. Amir, *Continuous functions' spaces with the bounded extension property,* Bull. Res. Council Israel Sect. F 10 (1962), 133–138. MR0143026 (26 #592)
3. S. A. Argyros, J. F. Castillo, A. S. Granero, M. Jiménez, and J. P. Moreno, *Complementation and embeddings of $c_0(I)$ in Banach spaces,* Proc. London Math. Soc. 85 (2002), 742–768. MR1936819 (2003k:46022)
4. S. A. Argyros and R. G. Haydon, *A hereditarily indecomposable L_∞-space that solves the scalar-plus-compact problem,* Acta Math. 206 (2011), 1–54. MR2784662 (2012e:46031)

5. J. W. Baker, *Some uncomplemented subspaces of $C(X)$ of the type $C(Y)$*, Studia Math. 36 (1970), 85–103. MR0275356 (43 #1113)
6. J. W. Baker, *Uncomplemented $C(X)$-subalgebras of $C(X)$*, Trans. Amer. Math. Soc. 186 (1973), 1–15. MR0331034 (48 #9369)
7. D. Blecher and T. Kania, *Finite generation in $C*$-algebras and Hilbert $C*$-modules,* in preparation.
8. N. Boudi, *Banach algebras in which every left ideal is countably generated,* Irish Math. Soc. Bull. 48 (2002), 17–24. MR1930521 (2003h:46069)
9. S. R. Caradus, W. E. Pfaffenberger and B. Yood, *Calkin Algebras and Algebras of Operators on Banach Spaces,* Lecture Notes in Pure Appl. Math. 9, Dekker, New York, 1974. MR0415345 (54 #3434)
10. H. B. Cohen, *Injective envelopes of Banach spaces,* Bull. Amer. Math. Soc. 70 (1964), 723–726. MR0184060 (32 #1536)
11. W. W. Comfort and S. Negrepontis, *The Theory of Ultrafilters,* Grundlehren Math. Wiss. 211, Springer, New York, 1974. MR0396267 (53 #135)
12. H. G. Dales, *Banach Algebras and Automatic Continuity,* London Math. Soc. Monogr. 24, Clarendon Press, Oxford, 2000. MR1816726 (2002e:46001)
13. H. G. Dales and W. Żelazko, *Generators of maximal left ideals in Banach algebras,* Studia Math. 212 (2012), 173–193. MR3008440
14. W. J. Davis, T. Figiel, W. B. Johnson, and A. Pełczyński, *Factoring weakly compact operators,* J. Funct. Anal. 17 (1974), 311–327. MR0355536 (50 #8010)
15. A. V. Ferreira and G. Tomassini, *Finiteness properties of topological algebras, I,* Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) 5 (1978), 471–488. MR0506996 (80d:46101)
16. L. Gillman, *Countably generated ideals in rings of continuous functions,* Proc. Amer. Math. Soc. 11 (1960), 660–666. MR0156189 (27 #6120)
17. M. González, *On essentially incomparable Banach spaces,* Math. Z. 215 (1994), 621–629. MR1269493 (95d:46012)
18. D. B. Goodner, *Projections in normed linear spaces,* Trans. Amer. Math. Soc. 69 (1950), 89–108. MR0037465 (12,266c)
19. W. T. Gowers and B. Maurey, *The unconditional basic sequence problem,* J. Amer. Math. Soc. 6 (1993), 851–874. MR1201238 (94k:46021)
20. B. Gramsch, *Eine Idealstruktur Banachscher Operatoralgebren,* J. Reine Angew. Math. 225 (1967), 97–115. MR0206727 (34 #6544)
21. A. S. Granero, *On complemented subspaces of $c_0(I)$*, Atti Sem. Mat. Fis. Univ. Modena Reggio Emilia 46 (1998), 35–36. MR1628581 (99c:46016)
22. N. Grønbæk, *Morita equivalence for Banach algebras,* J. Pure Appl. Algebra 99 (1995), 183–219. MR1327199 (96e:46099)
23. A. Grothendieck, *Une caractérisation vectorielle-métrique des espaces L^1*, Canad. J. Math. 7 (1955), 552–561. MR0076301 (17,877d)
24. N. Jacobson, *Basic Algebra II,* W. H. Freeman, San Francisco, 1980. MR0571884 (81g:00001)
25. T. Jech, *Set Theory, The Third Millennium Edition, Revised and Expanded,* Springer, Berlin, 2003. MR1940513 (2004g:03071)
26. R. V. Kadison and J. R. Ringrose, *Fundamentals of the Theory of Operator Algebras I: Elementary Theory,* Pure Appl. Math. 100, Academic Press, New York, 1983. MR0719020 (85j:46099)
27. R. V. Kadison and J. R. Ringrose, *Fundamentals of the Theory of Operator Algebras II: Advanced Theory,* Pure Appl. Math. 100, Academic Press, Orlando, FL, 1986. MR0859186 (88d:46106)
28. N. J. Kalton and N. T. Peck, *Twisted sums of sequence spaces and the three space problem,* Trans. Amer. Math. Soc. 255 (1979), 1–30. MR0542869 (82g:46021)

29. T. Kania and N. J. Laustsen, *Ideal structure of the algebra of bounded operators acting on a Banach space,* in preparation.
30. D. Kleinecke, *Almost-finite, compact, and inessential operators,* Proc. Amer. Math. Soc. 14 (1963), 863–868. MR0155197 (27 #5136)
31. P. Koszmider, *Banach spaces of continuous functions with few operators,* Math. Ann. 330 (2004), 151–183. MR2091683 (2005h:46027)
32. N. J. Laustsen, *Maximal ideals in the algebra of operators on certain Banach spaces,* Proc. Edinburgh Math. Soc. 45 (2002), 523–546. MR1933735 (2003i:47080)
33. N. J. Laustsen, *Commutators of operators on Banach spaces,* J. Operator Theory 48 (2002), 503–514. MR1962468 (2004c:47076)
34. J. Lindenstrauss, *On the extension property for compact operators,* Bull. Amer. Math. Soc. 68 (1962), 484–487. MR0139953 (25 #3377)
35. J. Lindenstrauss and L. Tzafriri, *Classical Banach Spaces I: Sequence Spaces,* Ergeb. Math. Grenzgeb. 92, Springer, Berlin, 1977. MR0500056 (58 #17766)
36. J. Lindenstrauss and L. Tzafriri, *Classical Banach Spaces II: Function Spaces,* Ergeb. Math. Grenzgeb. 97, Springer, Berlin, 1979. MR0540367 (81c:46001)
37. E. Luft, *The two-sided closed ideals of the algebra of bounded linear operators of a Hilbert space,* Czechoslovak Math. J. 18 (1968), 595–605. MR0238086 (38 #6362)
38. R. E. Megginson, *An Introduction to Banach Space Theory,* Grad. Texts in Math. 183, Springer, New York, 1998. MR1650235 (99k:46002)
39. B. S. Mityagin and I. S. Edelstein [I. S. Èdel'shteĭn], *Homotopy type of linear groups of two classes of Banach spaces,* Funct. Anal. Appl. 4 (1970), 221–231; transl. of: Funktsional. Anal. i Prilozhen. 4 (1970), no. 3, 61–72. MR0341526 (49 #6274d)
40. G. J. Murphy, *$C*$-algebras and Operator Theory,* Academic Press, Boston, 1990. MR1074574 (91m:46084)
41. L. Nachbin, *On the Hahn–Banach theorem,* An. Acad. Brasil. Ci. 21 (1949), 151–154. MR0031188 (11,114g)
42. A. Pełczyński, *On strictly singular and strictly cosingular operators. I. Strictly singular and strictly cosingular operators in $C(S)$-spaces,* Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 13 (1965), 31–36. MR0177300 (31 #1563)
43. A. Pietsch, *Operator Ideals,* North-Holland, Amsterdam, 1980. MR0582655 (81j:47001)
44. G. Plebanek, *A construction of a Banach space $C(K)$ with few operators,* Topology Appl. 143 (2004), 217–239. MR2081013 (2005j:46015)
45. B. Pospíšil, *On bicompact spaces,* Publ. Fac. Sci. Univ. Masaryk 270 (1939), 1–16. MR0001454 (1,241d)
46. R. T. Prosser, *On the ideal structure of operator algebras,* Mem. Amer. Math. Soc. 45 (1963). MR0151863 (27 #1846)
47. A. Rosenberg, *The number of irreducible representations of simple rings with no minimal ideals,* Amer. J. Math. 75 (1953), 523–530. MR0057477 (15,236f)
48. Th. Schlumprecht, *An arbitrarily distortable Banach space,* Israel J. Math. 76 (1991), 81–95. MR1177333 (93h:46023)
49. A. M. Sinclair and A. W. Tullo, *Noetherian Banach algebras are finite dimensional,* Math. Ann. 211 (1974), 151–153. MR0355607 (50 #8081)
50. A. Sobczyk, *Projection of the space (m) on its subspace (c_0),* Bull. Amer. Math. Soc. 47 (1941), 938–947. MR0005777 (3,205f)
51. E. L. Stout, *The Theory of Uniform Algebras,* Bogden and Quigley, Tarrytown-on-Hudson, NY, 1971. MR0423083 (54 #11066)
52. A. Szankowski, *$B(\mathcal{H})$ does not have the approximation property,* Acta Math. 147 (1981), 89–108. MR0631090 (83a:46033)
53. B. Yood, *Difference algebras of linear transformations on a Banach space,* Pacific J.

Math. 4 (1954), 615–636. MR0068117 (16,832e)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2920625 (Review) 43A10 43A20 46J10
Dales, H. G. [Dales, H. Garth] (4-LANC-MS); **Lau, A. T.-M.** (3-AB-MS);
Strauss, D. [Strauss, Dona Papert] (4-LEED-PM)
Second duals of measure algebras. (English summary)
Dissertationes Math. (Rozprawy Mat.) **481** (2012), 1–121.

The paper studies the second dual $M(G)''$ of the measure algebra of a locally compact group G when $M(G)''$ is equipped with the (first) Arens product extending the convolution on $M(G)$. As $M(G)'$ is a commutative von Neumann algebra (namely, the universal enveloping von Neumann algebra of $C_0(G)$), it may be identified with $C(\widetilde{G})$ where $\widetilde{G}$ is the spectrum of $M(G)'$. One may therefore identify $M(G)''$ with the measure space $M(\widetilde{G})$, and this is the viewpoint taken in the paper under review.

The space $\widetilde{G}$ is called the hyper-Stonean envelope of G. Of course one can define the hyper-Stonean envelope for any locally compact space, and the paper begins with a detailed discussion on Stonean and hyper-Stonean spaces with an emphasis on hyper-Stonean envelopes. An interesting sample result is the following: all uncountable, compact, metrisable spaces have the same hyper-Stonean envelope, for which the authors give a topological characterisation.

Let us then go through the main results of the paper concerning $M(G)''$. First, if G and H are two locally compact groups such that $M(G)''$ and $M(H)''$ are isometrically isomorphic, then G and H are topologically isomorphic (meaning that there is a group isomorphism between G and H that is also a homeomorphism). In other words, the Banach algebra $M(G)''$ determines G. The special case where G and H are compact is due to F. Ghahramani and J. P. McClure [Bull. London Math. Soc. **29** (1997), no. 2, 223–226; MR1426002 (98f:43001)]. (The paper under review includes a slightly shorter proof of this special case.)

Second, $\widetilde{G}$ is not closed under the multiplication of $M(G)'' \cong M(\widetilde{G})$ unless G is discrete. Here we identify $\widetilde{G}$ as a subset of $M(G)''$ using point masses in $M(\widetilde{G})$. This theorem is related to a result about $L^\infty(G)$ due to A. T. M. Lau, A. R. Medghalchi and J. S. Pym [J. London Math. Soc. (2) **48** (1993), no. 1, 152–166; MR1223900 (94k:43003)]: The spectrum of $L^\infty(G)$ is closed under the multiplication of $L^1(G)'' \cong L^\infty(G)'$ if and only if G is either discrete or compact. The authors first prove the theorem for $\mathbb{T}$, $\mathbb{R}$, $\mathbb{Z}_n^{\aleph_0}$ and the p-adic integers and then use inheritance properties and structural results to extend the theorem to all non-discrete locally compact groups. The authors also prove more refined results about the multiplication of points in $\widetilde{G}$. For example, a non-discrete G always admits two points in $\widetilde{G}$ that multiply to a continuous measure.

The paper then considers the topological centre problem for $M(G)''$, that is, whether the topological centre of $M(G)''$ consists merely of $M(G)$ (viewed through the canonical embedding). In [J. Funct. Anal. **224** (2005), no. 1, 217–229; MR2139110 (2006b:46063)], M. Neufang solved the problem for non-compact locally compact groups with non-measurable cardinality, so the most important case left out was that of compact groups. First the authors show that for any locally compact group G certain subsets of the spectrum of $L^\infty(G)$ determine the topological centre of $L^1(G)''$, which is known to be $L^1(G)$. (Note that the recent paper by T. Budak, N. Işık and Pym [Bull. Lond. Math. Soc. **43** (2011), no. 3, 495–506; MR2820139 (2012f:43003)] gives very strong results of this type when G is a *non-compact* locally compact group.) Then the authors offer some progress towards a solution for the topological centre problem for $M(G)''$ in the compact case. It should be mentioned that the general problem, which is very difficult, was recently solved by V. Losert et al. [“Proof of the Ghahramani-Lau conjecture”, preprint, http://www.math.yorku.ca/~steprans/Research/PDFSOfArticles/GLConj7.pdf].

The paper closes with eight open problems. *Pekka Salmi*

MR2945655 (Review) 22D12 20C15 22D30 46A40
de Jeu, Marcel (NL-LEID-MI); **Wortel, Marten** (NL-LEID-MI)
Positive representations of finite groups in Riesz spaces. (English summary)
Internat. J. Math. **23** (2012), *no. 7,* 1250076, 28 *pp.*

Many vector spaces appearing in the theory of unitary representations of locally compact groups have a structure of Banach lattices. Usually this structure is not explicitly studied. The authors undertake a systematic investigation of the representations of groups into Riesz spaces. A positive representation of a group G into a Riesz space E is a group homomorphism of G in the group of lattice automorphisms of E. Various notions of irreducibility for positive representations are considered. They prove that every positive representation of a finite group in a finite-dimensional Riesz space is a direct sum of irreducible representations. They also obtain an explicit description of the irreducible positive representations of finite groups. A notion of inducing is introduced, and analogs of the Frobenius reciprocity and the imprimitivity theorem are obtained. We refer to the paper for precise statements. Interesting examples like the regular representation of $\mathbb{Z}$ in l^∞ or in l^1 are briefly mentioned. *A. Derighetti*

References

1. C. D. Aliprantis and O. Burkinshaw, *Locally Solid Riesz Spaces with Applications to Economics* (American Mathematical Society, Providence, RI, 2003). MR2011364 (2005b:46010)
2. M. de Jeu and H. J. M. Messerschmidt, Jordan-Hölder theorems for Riesz spaces, to appear.
3. M. de Jeu and J. Rozendaal, Decomposing positive representations in L^p-spaces for Polish transformation groups, http://www.math.leidenuniv.nl/en/theses/240/, to appear.
4. J. Dixmier, *C^*-Algebras* (North-Holland Publishing, Amsterdam, 1977). MR0458185 (56 #16388)
5. S. Lang, *Algebra* (Springer, New York, 2002). MR1878556 (2003e:00003)
6. M. Larsen, On the conjugacy of element-conjugate homomorphisms. II, *Quart. J. Math. Oxford Ser.* (2) **47** (1996) 73–85. MR1380951 (97b:20070)
7. W. A. J. Luxemburg and A. C. Zaanen, *Riesz Spaces I* (North-Holland Publishing, Amsterdam, 1971).
8. P. Meyer-Nieberg, Banach Lattices (Springer, Berlin, 1991). MR1128093 (93f:46025)
9. H. H. Schaefer, *Banach Lattices and Positive Operators* (Springer, New York, 1974). MR0423039 (54 #11023)
10. J.-P. Serre, *Linear Representations of Finite Groups* (Springer, New York, 1977). MR0450380 (56 #8675)
11. B. Smit and H. W. Lenstra, Jr., Linearly equivalent actions of solvable groups, *J. Algebra* **228** (2000) 270–285. MR1760965 (2001f:20069)
12. L. A. Steen and J. A. Seebach, Jr., *Counterexamples in Topology* (Dover Publications, Mineola, NY, 1995). MR1382863 (96k:54001)
13. J. A. Wolf, *Harmonic Analysis on Commutative Spaces* (American Mathematical Society, Providence, RI, 2007). MR2328043 (2008f:22008)
14. A. C. Zaanen, *Introduction to Operator Theory in Riesz Spaces* (Springer, Derlin, 1997). MR1631533 (2000c:47074)

MR1367086 (97a:47068) 47D30 47B38
de Pagter, B. [de Pagter, Ben] (NL-DELF-TM);
Ricker, W. J. [Ricker, Werner J.] (5-NSW-SM)
Bicommutants of algebras of multiplication operators.
Proc. London Math. Soc. (3) **72** (1996), *no. 2,* 458–480.

FEATURED REVIEW.

The introduction of this paper describes its content so clearly and adequately that we cannot avoid the temptation of using parts of it for our report. Moreover, in writing this report we were in touch with the authors, and we use this opportunity to thank them for answering some of our questions and for sending us a preprint of a sequel to the paper under review. This sequel is entitled "Algebras of multiplication operators in Banach function spaces", and we will briefly mention its content later on.

We begin by reminding the reader of the classical bicommutant theorem due to von Neumann, which is a starting point of this work. According to this theorem, for each unital selfadjoint algebra $\mathcal{B}$ of operators on a Hilbert space, the bicommutant $\mathcal{B}^{cc}$ of $\mathcal{B}$ coincides with the closure of $\mathcal{B}$ in the weak operator topology τ_w. This result was an important part of von Neumann's program of the axiomatization of quantum physics. This and the intrinsic beauty of von Neumann's theorem have made the questions associated with this theorem an object of intense study in "pure" functional analysis. The general question of describing conditions on an operator algebra under which the bicommutant coincides with the closure of the algebra in the weak operator topology is far from being answered. It is known [D. Sarason, Pacific J. Math. **17** (1966), 511–517; MR0192365 (33 #590)] that the hypothesis of the selfadjointness of $\mathcal{B}$ is essential for the validity of von Neumann's theorem (even if the algebra $\mathcal{B}$ is commutative). This fact explains one of the serious problems arising when one wants to generalize this theorem to algebras of operators on Banach spaces: it is not clear which algebraic properties should replace selfadjointness.

On the other hand, the paper begins with some encouraging examples that suggest the possibility of extending the von Neumann theorem. We need some notation to describe these examples. The symbols $C[0,1]$ and $L^p[0,1]$, where $1 \le p \le \infty$, have their usual meaning, and we consider $C[0,1]$ as a subalgebra of $L^\infty[0,1]$. Each function $\varphi \in L^\infty$ generates a natural multiplication operator $M_\varphi\colon f \mapsto \varphi f$ which is bounded on each L^p. Following the authors we denote by $\mathfrak{M}_p$ the unital algebra $\{M_\varphi\colon\ \varphi \in C[0,1]\}$ of all such multiplication operators on L^p generated by the multipliers from $C[0,1]$. It is possible to verify that for each finite p the bicommutant $\mathfrak{M}_p^{cc}$ coincides with the algebra $\{M_\varphi\colon\ \varphi \in L^\infty[0,1]\}$ and that $\mathfrak{M}_\infty^{cc} = \{M_\varphi\colon\ \varphi \text{ is Riemann integrable}\}$.

These results (of which the latter is much more surprising) can certainly be considered as the desired extensions of von Neumann's theorem, and they lead the authors to the next more general question for the following algebras of multiplication operators. Replace $C[0,1]$ by an arbitrary closed subalgebra $\mathfrak{A}$ of $L^\infty(\mu)$, and consider in the space $\mathcal{L}(L^p)$ of all continuous operators on $L^p(\mu)$ the algebra $\mathfrak{M}_p(\mathfrak{A}) = \{M_\varphi\colon\ \varphi \in \mathfrak{A}\}$. The measure μ here can be any non-pathological measure; to be exact, the authors deal with the class of Maharam measures. What is the bicommutant of this algebra? The purpose of this work is to give a complete answer to this question. At this point it is important to mention that this answer depends heavily on whether p is finite or not, and also on whether one deals with real or complex algebras. The first distinction is more important, as the solution for infinite p was hard to guess, while the finite p case has its "antecedents", at least for the algebra $L^\infty(\mu)$ over a σ-finite measure (for $p = 2$ this is Theorem IX.6.6 in J. B. Conway's book, *A course in functional analysis* [Springer, New York, 1985; MR0768926 (86h:46001)], and for other finite p the same proof can easily be adapted). We will confine our attention mostly to spaces over the reals, mentioning the complex case only briefly. Theorem 1.1 handles the case when p is finite and states that $\mathfrak{M}_p(\mathfrak{A})^{cc} = \tau_w - \mathrm{cl}(\mathfrak{M}_p(\mathfrak{A}_1))$, where $\mathfrak{A}_1$ denotes the smallest unital subalgebra of $L^\infty(\mu)$ containing $\mathfrak{A}$.

To formulate the answer for $p = \infty$ we need an additional concept of upper and lower elements. Namely, if $\mathfrak{U}$ is a unital subalgebra of $L^\infty(\mu)$ and $f \in L^\infty(\mu)$ then consider $f^\uparrow = \sup\{u \in \mathfrak{U}\colon\ u \le f\}$ and $f^\downarrow = \inf\{u \in \mathfrak{U}\colon\ f \le u\}$, where the sup and inf above are taken in the Dedekind complete vector lattice $L^\infty(\mu)$ and should not be confused with the pointwise sup and inf. The collection $\mathfrak{R}(\mathfrak{U}) = \{f \in L^\infty\colon\ f^\uparrow = f^\downarrow\}$ is called the Dedekind closure and was introduced by S. Kaplan. The Dedekind closure is a closed unital subalgebra of $L^\infty(\mu)$ and this object is crucial for formulating the answer for $p = \infty$ in Theorem 1.2, which states that $\mathfrak{M}_\infty(\mathfrak{A})^{cc} = \mathfrak{M}_\infty(\mathfrak{R}(\mathfrak{A}_1))$. In the case of the complex field, the algebra $\mathfrak{A}_1$ in the statement of Theorem 1.1 should be replaced by a "selfadjoint" hull $\mathfrak{A}^\# = \{\sum f_j \overline{g}_j\colon\ f_j, g_j \in \mathfrak{A}\}$ and in the statement of Theorem 1.2 by a naturally defined Dedekind closure of $\mathfrak{A}^\#$ in $L^\infty_{\mathbf{C}}(\mu)$.

Section 2 contains many well-chosen examples illustrating the main results, the proofs of which are given in Sections 3 and 4. The concluding section, Section 5, has further examples and relevant remarks. A few words about the proofs. They are rather involved but neatly presented, and show that the problem at hand has a much more intimate relationship with the theory of vector and Banach lattices than one might suspect. The main ingredients of the proofs are the operator of the conditional expectation and a detailed analysis of the structure of the Dedekind closure.

The moment one starts reading through the proofs the question arises as to what extent the concrete geometry of L^p spaces is essential for the results obtained in the paper. The authors themselves answer this question in the above-mentioned preprint, in which—among many other things—they extend the results of the paper under review to a much more general class of Banach spaces, in particular to (L^1, L^∞) interpolation (fully symmetric) Banach function spaces with order continuous norm. Notice that from an example by J. Dieudonné [Portugal. Math. **14** (1955), 35–38; MR0078667 (17,1228b)] it follows that the interpolation condition above is essential. For the details, more connections and, of course, the proofs, we refer the reader to these interesting papers. *Yu. A. Abramovich* and *Arkady K. Kitover*

MR1716969 (2000j:47120) 47L10 46E40 47B38
de Pagter, B. [de Pagter, Ben] (NL-DELF-CS);
Ricker, W. J. [Ricker, Werner J.] (5-NSW-SM)
Algebras of multiplication operators in Banach function spaces. (English summary)
J. Operator Theory **42** (1999), *no. 2,* 245–267.

Let E be a Banach function space on a Maharam measure space and $\mathfrak{A}$ a subalgebra of L^∞. Let $\mathfrak{M} = \{M_\varphi\colon\ \varphi \in \mathfrak{A}\}$ be the corresponding algebra of multiplication operators on E. The authors consider the weak operator closure $\overline{\mathfrak{M}}$, the bicommutant $\mathfrak{M}^{cc}$ (both taken in $\mathcal{L}(E)$), and $\mathfrak{M}(\overline{\mathfrak{A}})$, where $\overline{\mathfrak{A}}$ is the $\mathfrak{T}_s(L^\infty, L^1)$-closure of $\mathfrak{A}$ in L^∞. They show that $\overline{\mathfrak{M}} \subseteq \mathfrak{M}^{cc}$ and that if $E \subseteq L^1 + L^\infty$ is fully symmetric (in the sense that $\|f\|_E \leq \|g\|_E$ whenever $\int_0^t f^* \leq \int_0^t g^*$ for every $t > 0$, where f^* is the decreasing rearrangement of f), then $\mathfrak{M}^{cc} \subseteq \mathfrak{M}(\overline{\mathfrak{A}})$. This extends an earlier result by the same authors concerning L^p spaces [Proc. London Math. Soc. (3) **72** (1996), no. 2, 458–480; MR1367086 (97a:47068)]; the case $p = 2$ is a form of von Neumann's bicommutant theorem. The paper is carefully and elegantly written and contains full descriptions of some useful examples.

D. H. Fremlin

References

1. C.D. Aliprantis, O. Burkinshaw, *Positive Operators*, Academic Press, Orlando 1985. MR0809372 (87h:47086)
2. C. Bennett, R. Sharpley, *Interpolation of Operators*, Academic Press, New York 1988. MR0928802 (89e:46001)
3. F.F. Bonsall, J. Duncan, *Complete Normed Algebras*, Ergeb. Math. Grenzgeb., vol. 80, Springer-Verlag, Berlin-New York-Heidelberg 1973. MR0423029 (54 #11013)
4. J. Dieudonné, Sur la bicommutante d'une algèbre d'opérateurs, *Portugaliae Math.* **14**(1955), 35–38. MR0078667 (17,1228b)
5. H.R. Dowson, *Spectral Theory of Linear Operators*, London Math. Soc. Monographs (N.S.), vol 12, Academic Press, London 1978. MR0511427 (80c:47022)
6. D.H. Fremlin, Stable subspaces of $L^1 + L^\infty$, *Math. Proc. Cambridge Philos. Soc.* **64**(1968), 625–643. MR0225154 (37 #749)
7. D.H. Fremlin, *Topological Riesz Spaces and Measure Theory*, Cambridge Univ. Press, Cambridge 1974. MR0454575 (56 #12824)
8. T.A. Gillespie, Factorization in Banach function spaces, *Indag. Math.* **43**(1981), 287–300. MR0632168 (82k:46047)
9. D. Hadwin, M. Orhon, Reflexivity and approximate reflexivity for bounded Boolean algebras of projections, *J. Funct. Anal.* **87**(1989), 348–358. MR1026857 (91e:47047)
10. A. Ionescu Tulcea, C. Ionescu Tulcea, *Topics in the Theory of Lifting*, Ergeb. Math. Grenzgeb, vol. 48, Springer-Verlag, Heidelberg 1969. MR0276438 (43 #2185)
11. S. Kaplan, *The Bidual of $C(X)$*. I, North-Holland Math. Stud., vol. 101, Amsterdam 1985. MR0776606 (86k:46001)
12. Y. Katznelson, *An Introduction to Harmonic Analysis*, Dover Publications, New York 1968. MR0422992 (54 #10976)
13. S.G. Krein, Ju.I. Petunin, E.M. Semenov, *Interpolation of Linear Operators*, Transl. Math. Monographs, vol. 54, Amer. Math. Soc., Providence 1982. MR0649411 (84j:46103)

14. Z. Lipecki, Extension of vector lattice homomorphisms revisted, *Indag. Math.* **47** (1985), 229–233. MR0799083 (86i:46008)
15. G.Ya. Lozanovskii, On some Banach lattices, *Siberian Math. J.* **10**(1969), 419–431.
16. W.A.J. Luxemburg, A.R. Schep, An extension theorem for Riesz homomorphisms, *Indag. Math.* **41**(1979), 145–154. MR0535562 (80i:47051)
17. P. Meyer-Nieberg, *Banach Lattices*, Springer-Verlag, New York-Heidelberg-Berlin 1991. MR1128093 (93f:46025)
18. B. de Pagter, W.J. Ricker, Bicommutants of algebras of multiplication operators, *Proc. London Math. Soc. (3)* **79**(1996), 458–480. MR1367086 (97a:47068)
19. S. Reisner, On two theorems of Lozanovskii concerning intermediate Banach lattices, in *Geometric Aspects Funct. Anal.*, Lecture Notes in Math., vol. 1317, Springer-Verlag, Berlin-Heidelberg-New York 1988, pp. 67–83. MR0950976 (89j:46019)
20. A.C. Zaanen, *Integration*, North Holland, Amsterdam 1967. MR0222234 (36 #5286)
21. A.C. Zaanen, *Riesz Spaces*. II, North Holland, Amsterdam 1983. MR0704021 (86b:46001)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2363429 (2009e:46037) 46G10 46B42 47A67 47B40 47B60
de Pagter, B. [de Pagter, Ben] (NL-DELF-IAM);
Ricker, W. J. [Ricker, Werner J.] (D-EICHM)
$C(K)$-representations and R-boundedness. (English summary)
J. Lond. Math. Soc. (2) **76** (2007), *no. 2,* 498–512.

Let K be a compact Hausdorff space and X be a Banach space. The authors study continuous representations $\Phi\colon \mathcal{C}(K)\to\mathcal{L}(X)$, where $\mathcal{C}(K)$ stands for the space of all continuous functions from K to $\mathbb{C}$.

In this framework, it is useful to find a regular σ-additive spectral measure $P\colon \mathcal{B}(K)\to\mathcal{L}(X)$, where $\mathcal{B}(K)$ denotes the σ-algebra of Borel subsets of K, such that

$$\Phi(f)=\int_K f\,dP \quad \text{for every } f\in\mathcal{C}(K).$$

The first main result of the paper asserts that such a P does exist, whenever the set $\{\Phi(f)\colon f\in\mathcal{C}(K), \|f\|\le 1\}$ is R-bounded in $\mathcal{L}(X)$.

The proof partly relies on results concerning Banach lattices that are useful in studying the orbits

$$\{\Phi(f)(x)\colon\ f\in\mathcal{C}(K)\},$$

where $x\in X$ is fixed. Conditions are given on X ensuring that the R-boundedness condition is automatically satisfied.

These results are applied to prove that, if $\Phi_1\colon\mathcal{C}(K_1)\to\mathcal{L}(X)$ and $\Phi_2\colon\mathcal{C}(K_2)\to\mathcal{L}(X)$ are two commuting continuous representations, and if, say, Φ_1 is R-bounded, then there exists a continuous representation $\Phi\colon\mathcal{C}(K_1\times K_2)\to\mathcal{L}(X)$, which extends Φ_1 and Φ_2 in an obvious sense.

This result is an analogue of a result obtained by T. A. Gillespie [J. Funct. Anal. **148** (1997), no. 1, 70–85; MR1461494 (98h:47048)], and the proof uses some arguments due to Gillespie.
Richard Becker

References

1. Y. A. Abramovich, E. L. Arenson and A. K. Kitover, *Banach $C(K)$-modules and operators preserving disjointness,* Pitman Research Notes in Mathematics Series 277 (Longman Scientific & Technical, Harlow, Essex, 1992). MR1202880 (94d:47027)
2. W. Arendt and S. Bu, 'The operator-valued Marcinkiewicz multiplier theorem and maximal regularity', *Math. Z.* 240 (2002) 311–343. MR1900314 (2003i:42016)
3. S. A. Argyros and A. Tolias, 'Methods in the theory of hereditarily indecomposable Banach spaces', *Mem. Amer. Math. Soc.* 170 (2004). MR2053392 (2005f:46022)
4. E. Berkson and T. A. Gillespie, 'Spectral decompositions and harmonic analysis in UMD spaces', *Studia Math.* 112 (1994) 13–49. MR1307598 (96c:42022)
5. J. Bourgain, 'Some remarks on Banach spaces in which martingale differences are unconditional', *Ark. Mat.* 21 (1983) 163–168. MR0727340 (85a:46011)
6. P. Clément, B. de Pagter, F. A. Sukochev and H. Witvliet, 'Schauder decompositions and multiplier theorems', *Studia Math.* 138 (2000) 135–163. MR1749077 (2002c:47036)
7. P. Clément and J. Prüss, 'An operator-valued transference principle and maximal regularity on vector-valued L_p-spaces', *Evolution equations and their applications in physical and life sciences* (ed. G. Lumer and L. Weis; Marcel Dekker, New

York, 2001) 67–87. MR1816437 (2001m:47064)

8. R. Denk, M. Hieber and J. Prüss, 'R-boundedness and problems of elliptic and parabolic type', *Mem. Amer. Math. Soc.* 166 (2003). MR2006641 (2004i:35002)
9. J. Diestel, H. Jarchow and A. Tonge, *Absolutely summing operators,* Cambridge Studies in Advanced Mathematics 43 (Cambridge University Press, Cambridge, 1995). MR1342297 (96i:46001)
10. H. R. Dowson, *Spectral theory of linear operators* (Academic Press, London, 1978). MR0511427 (80c:47022)
11. N. Dunford and J. T. Schwartz, *Linear operators I: general theory* (Wiley-Interscience, New York, 1958). MR0117523 (22 #8302)
12. N. Dunford and J. T. Schwartz, *Linear operators III: spectral operators* (Wiley-Interscience, New York, 1971). MR1009164 (90g:47001c)
13. V. Ferenczi, 'A uniformly convex hereditarily indecomposable Banach space', *Israel J. Math.* 102 (1997) 199–225. MR1489106 (98m:46013)
14. T. A. Gillespie, 'Boundedness criteria for Boolean algebras of projections', *J. Funct. Anal.* 148 (1997) 70–85. MR1461494 (98h:47048)
15. W. T. Gowers and B. Maurey, 'The unconditional basic sequence problem', *J. Amer. Math. Soc.* 6 (1993) 851–874. MR1201238 (94k:46021)
16. M. Hoffmann, N. J. Kalton and T. Kucherenko, 'R-bounded approximating sequences and applications to semigroups', *J. Math. Anal. Appl.* 294 (2004) 373–386. MR2061331 (2005e:46034)
17. N. J. Kalton and L. Weis, 'The H^∞-calculus and sums of closed operators', *Math. Ann.* 321 (2001) 319–345. MR1866491 (2003a:47038)
18. I. Kluvánek, 'Characterization of Fourier–Stieltjes transforms of vector and operator valued measures', *Czech. Math. J.* 17 (1967) 261–276. MR0230872 (37 #6430)
19. B. Maurey and G. Pisier, 'Séries de variables aléatoires vectorielles indépendantes et propriétés géométriques des espaces de Banach', *Studia Math.* 58 (1976) 45–90. MR0443015 (56 #1388)
20. P. Meyer-Nieberg, *Banach lattices* (Springer, Berlin, 1991). MR1128093 (93f:46025)
21. B. de Pagter and W. J. Ricker, 'Products of commuting Boolean algebras of projections and Banach space geometry', *Proc. London Math. Soc.* (3) 91 (2005) 483–508. MR2167095 (2007a:47089)
22. G. Pisier, 'Some results on Banach spaces without local unconditional structure', *Compos. Math.* 37 (1978) 3–19. MR0501916 (80e:46012)
23. W. J. Ricker, 'Spectral operators of scalar-type in Grothendieck spaces with the Dunford–Pettis property', *Bull. London Math. Soc.* 17 (1985) 268–270. MR0806430 (87e:47041)
24. W. J. Ricker, 'Well bounded operators of type (B) in H.I. spaces', *Acta Sci. Math. (Szeged)* 59 (1994) 475–488. MR1317168 (96j:47027)
25. W. J. Ricker, *Operator algebras generated by commuting projections: a vector measure approach,* Lecture Notes in Mathematics 1711 (Springer, Berlin, 1999). MR1718704 (2001b:47055)
26. L. Weis, 'A new approach to maximal L_p-regularity', *Evolution equations and their applications in physical and life sciences* (ed. G. Lumer and L. Weis; Marcel Dekker, New York, 2001) 195–214. MR1818002 (2002a:47068)
27. H. Witvliet, 'Unconditional Schauder decompositions and multiplier theorems', PhD Thesis, Delft University of Technology, 2000.
28. A. C. Zaanen, *Riesz spaces II* (North-Holland, Amsterdam, 1983). MR0704021 (86b:46001)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR937631 (89d:47079) 47B55 46B30 47H09
de Pagter, B. [de Pagter, Ben] (1-CAIT); **Schep, A. R.** (1-SC)
Measures of noncompactness of operators in Banach lattices.
J. Funct. Anal. **78** (1988), *no. 1,* 31–55.

Let E be complex Banach lattice. A subset D of E is said to be almost order bounded if for every $\varepsilon > 0$ there exists $0 \le u \in E$ such that $\|(|f| - u)^+\| \le \varepsilon$ for all $f \in D$. For a norm bounded subset D of E let $\rho(D) := \inf\{\delta > 0 \colon (\exists 0 \le u \in E)\ (\forall f \in D) \|(|f| - u)^+\| \le \delta\}$. The ball measure of noncompactness of D will be denoted by $\beta(D)$. A subset $D \subset E$ is said to be PL-compact whenever D is p_φ-precompact for every $0 \le \varphi \in E^*$, where $p_\varphi(x) := \langle |x|, \varphi\rangle$, $x \in E$. It is shown that $\beta(D) = \rho(D)$ for PL-precompact subsets of D. If T is an order bounded linear operator from a Banach lattice E into a Banach lattice F, then its measure of non-semicompactness is defined by $\rho(T) := \inf\{k \ge 0 \colon \rho(TD) \le k\rho(D)$ for all norm bounded $D \subset E\}$. An order bounded operator $T \in \mathcal{L}_b(E, F)$ is said to be AM-compact whenever T maps order bounded sets into relatively compact sets. For AM-compact operators it is shown that $\beta(T) = \rho(T)$. With respect to the essential spectral radius $r_{\rm ess}(T) := \lim_{n\to\infty}(\beta(T)^n)^{1/n}$ the authors show that if $0 \le S \le T$ and S is AM-compact, then $r_{\rm ess}(S) \le r_{\rm ess}(T)$. If E is a Banach function space with order continuous norm, then the same result holds if one assumes S to be an integral operator.

Finally, we mention that if the dual E^* is nonatomic, then every norm bounded disjointness preserving operator T of E into E has the property that $r_{\rm ess}(T) = r(T)$. For further results concerning other classes of operators such as the Maharam operators we refer to this interesting paper. *W. A. J. Luxemburg*

MR1724533 (2000i:47104) 47H10 45G10 47J05 47N20
Dhage, B. C. (6-MAGA-MR)
Fixed point theorems in ordered Banach algebras and applications. **(English summary)**
Panamer. Math. J. **9** (1999), *no. 4,* 83–102.

Let X be a real Banach algebra and let K be a normal cone in X such that $KK \subset K$, where $KK = \{xy\colon\ x, y \in K\}$. Assume that A and B are two operators defined on a segment in X and satisfying some regularity type conditions (for example, A is continuous and Lipschitzian, B is continuous, bounded and A, B are monotone with respect to the order determined by a cone K). Then the equation $A(x)B(x) = x$ has a solution in X which can be obtained via the procedure of successive approximation. Similar results are proved in other settings (for example, B is completely continuous) with help of the technique associated with measures of noncompactness. Application to a nonlinear integral equation is also given. *Józef Banaś*

MR2590616 (2011a:46072) 46H15
Dirksen, Sjoerd (NL-DELF-IAM); **de Jeu, Marcel** (NL-LEID-MI);
Wortel, Marten (NL-LEID-MI)

Extending representations of normed algebras in Banach spaces. (English summary)

Operator structures and dynamical systems, 53–72, *Contemp. Math.*, 503, *Amer. Math. Soc.*, *Providence, RI*, 2009.

Let A be a normed algebra. The authors study the possibility of extending a given Banach representation of an ideal $J \subset A$ to all of A. Additionally, given a Banach representation π of A, they study the possibility of defining representations of the various centralizer algebras of A which are compatible with π. In particular: (1) Let X be a Banach space, whose space of bounded linear operators is denoted by $\mathcal{B}(X)$, and let $J \subset A$ be an ideal which has a bounded left approximate identity. If $\pi\colon J \to \mathcal{B}(X)$ is a nondegenerate representation, then π extends uniquely to a representation of A. (2) If A has a bounded left approximate identity, and if $\pi\colon A \to \mathcal{B}(X)$ is a nondegenerate faithful representation which is a topological algebra isomorphism of A and $\pi(A)$, then A and its double centralizer algebra $\mathcal{M}(A)$ embed canonically into the left centralizer algebra $\mathcal{M}_l(A)$ of A. Within identification, one has $A \subset \mathcal{M}(A) \subset \mathcal{M}_l(A)$, and π extends uniquely to a representation $\overline{\pi}\colon \mathcal{M}_l(A) \to \mathcal{B}(X)$; $\overline{\pi}$ is an isomorphism of topological algebras between $\mathcal{M}_l(A)$ and the left normalizer of $\pi(A)$ in $\mathcal{B}(X)$. There are special-case results when A is ordered, or when A has a bounded involution, X is Hilbert, and π is involutive. The right-sided versions of the results are also developed, since they are not symmetric with the left.

{For the entire collection see MR2590612 (2010g:46082)}

David A. Robbins

MR1135978 (93d:46073) 46H05 46H25 46J45
Dixon, P. G. (4-SHEF)
Topologically nilpotent Banach algebras and factorisation.
Proc. Roy. Soc. Edinburgh Sect. A **119** (1991), *no. 3-4,* 329–341.

For a Banach algebra A, let $N_A(n)$ denote the supremum of all $\|x_1x_2\cdots x_n\|^{1/n}$ for $\|x_i\|\le 1$, and $S_A(N)$ the supremum of all $\|x^n\|^{1/n}$ with $\|x\|\le 1$. The algebra is said to be topologically nilpotent if $\lim N_A(n)=0$, and is said to be uniformly topologically nil if $\lim S_A(n)=0$. In the present paper, the author starts by giving a quantitative version of the fundamental result of J. K. Miziołek, T. Müldner and A. Rek [Studia Math. **43** (1972), 41–50; MR0306909 (46 #6030)] that these two concepts are equivalent for commutative algebras. For certain ordered Banach algebras, he then shows that a weaker condition on the sequence $\|x^n\|^{1/n}$ is equivalent to A being uniformly topologically nil. The main result of the present paper is that if $S_A(n)\to 0$ rapidly enough, then, even in the noncommutative case, A is topologically nilpotent. In a note added in proof, the author mentions that V. Müller has constructed a uniformly topologically nil Banach algebra which is not topologically nilpotent. This example is part of an interesting joint paper of the author and Müller which will appear in Studia Math.

In their 1972 paper, Miziołek et al. observed that no topologically nilpotent algebra can have a bounded approximate identity. Because of this result, much of the subsequent research has emphasized the incompatibility of topological nilpotence with various factorization properties. In the present paper, the author shows that A being topologically nilpotent precludes $AX=X$ for any nonzero Banach module. *Sandy Grabiner*

MR570888 (81g:47037) 47B55
Dodds, P. G.; **Fremlin, D. H.**
Compact operators in Banach lattices.
Israel J. Math. **34** (1979), *no. 4,* 287–320 (1980).

The principal result is that each positive operator from a Banach lattice E into a Banach lattice F which has a compact majorant is itself compact provided the norms of E' and F are order continuous. In order to obtain this theorem the authors introduce the class of AMAL-compact operators $T \in \mathcal{L}(E, F)$ for which all bicompositions $j_{v'} \circ T \circ j_u$ are compact for every $0 \le u \in E$, $0 \le v' \in F'$ and the canonical injections $j_u \colon E_u \to E$ and $j_{v'} \colon F \to (F, v')$ [H. H. Schaefer, *Banach lattices and positive operators*, see p. 260, Springer, New York, 1974; MR0423039 (54 #11023)]. The paper contains also a very extensive investigation of related classes of operators such as kernel operators, Dunford-Pettis operators and their order and compactness properties. It may be added that recently the main result has been extended considerably by C. D. Aliprantis and O. Burkinshaw [Math. Z. **174** (1980), no. 3, 289–298]. *R. J. Nagel*

MR1744698 (2001i:47062) 47B65 46B42 47D06
Drnovšek, Roman (SV-LJUBMP)
Triangularizing semigroups of positive operators on an atomic normed Riesz space. (English summary)
Proc. Edinburgh Math. Soc. (2) **43** (2000), *no. 1,* 43–55.

Beginning with the well-known work of T. Andô [J. Fac. Sci. Hokkaido Univ. Ser. I. **13** (1957), 214–228; MR0092127 (19,1067b)] and its generalization by H.-J. Krieger [*Beiträge zur Theorie positiver Operatoren*, Akademie Verlag, Berlin, 1969; MR0415403 (54 #3491)] there have been a number of results implying the existence of invariant subspaces for positive operators on Banach lattices; a good summary is contained in the expository paper of Y. A. Abramovich, C. D. Aliprantis and O. Burkinshaw [Rend. Istit. Mat. Univ. Trieste **29** (1998), suppl., 3–79 (1999); MR1696022 (2000f:47062)]. Some special cases of the Ando-Krieger theorem were extended to semigroups of quasinilpotent operators by M. D. Choi et al. [Indiana Univ. Math. J. **42** (1993), no. 1, 15–25; MR1218704 (94e:47009)].

The paper under review contains several interesting results on the existence of invariant subspaces (in fact, invariant ideals) for certain semigroups of operators on normed Riesz spaces. One theorem is the following: A multiplicative semigroup of positive operators on a normed Riesz space has a nontrivial invariant closed ideal if there is a positive atom in the space such that each of the operators in the semigroup is quasinilpotent at the atom. If the Riesz space is atomic and the operators in the semigroup are quasinilpotent at each of the atoms in a maximal orthogonal system of positive atoms, then it is shown that there is a chain of invariant bands that is maximal in the lattice of all bands. Also, a positive operator that is not "ideal-triangularizable" is constructed on a Banach lattice that contains an atom (by perturbing a well-known example of Schaefer), providing a counterexample to a question of M. T. Jahandideh [Proc. Amer. Math. Soc. **125** (1997), no. 9, 2661–2670; MR1396983 (97j:47052)]. *P. Rosenthal*

References

1. Y. A. Abramovich, C. D. Aliprantis and O. Burkinshaw, On the spectral radius of positive operators, *Math. Z.* **211** (1992), 593–607. MR1191098 (94a:47064)
2. Y. A. Abramovich, C. D. Aliprantis and O. Burkinshaw, Invariant subspaces of operators on l_p-spaces, *J. Funct. Analysis* **115** (1993), 418–424. MR1234398 (94h:47009)
3. Y. A. Abramovich, C. D. Aliprantis and O. Burkinshaw, Invariant subspace theorems for positive operators, *J. Funct. Analysis* **124** (1994), 95–111. MR1284604 (95e:47006)
4. Y. A. Abramovich, C. D. Aliprantis and O. Burkinshaw, The invariant subspace problem: some recent advances, *Rend. Istit. Mat. Univ. Trieste Suppl.* **29** (1998), 1–76. MR1696022 (2000f:47062)
5. C. D. Aliprantis and O. Burkinshaw, *Positive operators* (Academic Press, Orlando, 1985). MR0809372 (87h:47086)
6. G. Birkhoff, *Lattice theory*, AMS Colloq. Publ., vol. 25 (Providence, RI, 1967). MR0227053 (37 #2638)
7. M. D. Choi, E. A. Nordgren, H. Radjavi, P. Rosenthal and Y. Zhong, Triangularizing semigroups of quasinilpotent operators with non-negative entries, *Indiana Univ. Math. J.* **42** (1993), 15–25. MR1218704 (94e:47009)

8. M. T. Jahandideh, On the ideal-triangularizability of positive operators on Banach lattices, *Proc. Am. Math. Soc.* **125** (1997), 2661–1670. MR1396983 (97j:47052)
9. M. T. Jahandideh, Positive operators with p-hyperinvariant closed ideals. Preprint.
10. W. A. J. Luxemburg and A. C. Zaanen, *Riesz spaces I* (North-Holland, Amsterdam, 1971). MR0511676 (58 #23483)
11. P. Meyer-Nieberg, *Banach lattices* (Springer, 1991). MR1128093 (93f:46025)
12. E. Nordgren, H. Radjavi and P. Rosenthal, Triangularizing semigroups of compact operators, *Indiana Univ. Math. J.* **33** (1984), 271–275. MR0733900 (85b:47047)
13. H. Radjavi, On reducibility of semigroups of compact operators, *Indiana Univ. Math. J.* **39** (1990), 499–515. MR1089051 (91m:47009)
14. J. R. Ringrose, *Compact non-self-adjoint operators* (Van Nostrand Reinhold Math. Studies, London, 1971).
15. H. H. Schaefer, *Banach lattices and positive operators*, Grundlehren Math. Wiss. Bd. 215 (Springer, 1974). MR0423039 (54 #11023)
16. H. H. Schaefer, Topologische Nilpotenz irreduzibler Operatoren, *Math. Z.* **117** (1970), 135–140. MR0276802 (43 #2542)
17. A. C. Zaanen, *Riesz spaces II* (North-Holland, Amsterdam, 1983). MR0704021 (86b:46001)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1818060 (2001m:47012) 47A15 47B60 47D03
Drnovšek, Roman (SV-LJUBMP)
Common invariant subspaces for collections of operators. (English summary)
Integral Equations Operator Theory **39** (2001), *no. 3,* 253–266.

The author uses joint spectral radius and the Lomonosov-Hilden technique to prove the following theorem: Let X be a Banach space of dimension at least two and $\mathcal{C}$ a collection of bounded operators on X. If $\mathcal{C}$ is finitely quasinilpotent at a nonzero vector $x_0 \in X$ (i.e. for any finite subset $\mathcal{F}$ of $\mathcal{C}$, the joint spectral radius of $\mathcal{F}$ at x_0 is 0) and $\mathcal{C}$ contains a nonzero compact operator, then $\mathcal{C}$ and $\mathcal{C}'$ (the commutant of $\mathcal{C}$) have a common nontrivial invariant subspace.

The author uses the Lomonosov-Hilden technique again to obtain the following Banach lattice analogue: Let E be a Banach lattice of dimension at least two and $\mathcal{C}$ a collection of bounded positive operators on E. If $\mathcal{C}$ is finitely quasinilpotent at a nonzero positive vector and some member of $\mathcal{C}$ dominates some nonzero AM-compact operator then $\mathcal{C}$ and $\mathcal{C}^\sharp$ ($\mathcal{C}^\sharp = \{C \in X\colon TC \le CT \text{ for all } T \in \mathcal{C}\}$) have a common nontrivial closed ideal.

Combining this result with the result of Y. V. Turovskiĭ [J. Funct. Anal. **162** (1999), no. 2, 313–322; MR1682061 (2000d:47017)] that multiplicative semigroups of quasinilpotent operators are finitely quasinilpotent, the author generalizes B. de Pagter's theorem [Math. Z. **192** (1986), no. 1, 149–153; MR0835399 (87d:47052)] to semigroups and shows that a multiplicative semigroup of quasinilpotent compact positive operators on a Banach lattice of dimension at least two has a nontrivial invariant closed ideal.

Gordon Wilson MacDonald

References

1. Y.A. Abramovich, C.D. Aliprantis, O. Burkinshaw, *Invariant subspaces of operators on l_p spaces*, J. Funct. Anal. **115** (1993), 418–424. MR1234398 (94h:47009)
2. Y.A. Abramovich, C.D. Aliprantis, O. Burkinshaw, *Invariant subspace theorems for positive operators*, J. Funct. Anal. **124** (1994), 95–111. MR1284604 (95e:47006)
3. Y.A. Abramovich, C.D. Aliprantis, O. Burkinshaw, *On the spectral radius of positive operators*, Math. Z. **211** (1992), 593–607. MR1191098 (94a:47064)
4. Y.A. Abramovich, C.D. Aliprantis, O. Burkinshaw, *The invariant subspace problem: some recent advances*, Rend. Istit. Mat. Univ. Trieste Suppl. **29** (1998), 1–76. MR1696022 (2000f:47062)
5. C.D. Aliprantis, O. Burkinshaw, *Positive operators*, Academic Press, Orlando 1985. MR0809372 (87h:47086)
6. M.D. Choi, E.A. Nordgren, H. Radjavi, P. Rosenthal, Y. Zhong, *Triangularizing semigroups of quasinilpotent operators with non-negative entries*, Indiana Univ. Math. J. **42** (1993), 15–25. MR1218704 (94e:47009)
7. G. Cigler, R. Drnovšek, D. Kokol-Bukovšek, T. Laffey, M. Omladič, H. Radjavi, P. Rosenthal, *Invariant subspaces for semigroups of algebraic operators*, J. Funct. Anal. **160** (1998), 452–465. MR1665294 (2000b:47015)
8. R. Drnovšek, *On reducibility of semigroups of compact quasinilpotent operators*, Proc. Amer. Math. Soc. **125** (1997), 2391–2394. MR1422865 (97m:47007)
9. R. Drnovšek, *Triangularizing semigroups of positive operators on an atomic normed Riesz space*, Proc. Edinburgh Math. Soc. **43** (2000), 43–55. MR1744698

(2001i:47062)
10. R. Drnovšek, D. Kokol-Bukovšek, L. Livshits, G. MacDonald, M. Omladič, H. Radjavi, *An irreducible semigroup of non-negative square-zero operators*, preprint. cf. MR 2003j:47051
11. D. Hadwin, E. Nordgren, M. Radjabalipour, H. Radjavi and P. Rosenthal, *A nil algebra of bounded operators on Hilbert space with semisimple norm closure*, Integral Equat. Oper. Th. **9** (1986), 739–743. MR0860869 (87k:47104)
12. D. Hadwin, E. Nordgren, M. Radjabalipour, H. Radjavi, P. Rosenthal, *On simultaneous triangularization of collections of operators*, Houston J. Math. **17** (1991), no. 4, 581–602. MR1147275 (93a:47004)
13. M. T. Jahandideh, *On the ideal-triangularizability of positive operators on Banach lattices*, Proc. Amer. Math. Soc. **125** (1997), 2661–1670. MR1396983 (97j:47052)
14. M. Lambrou, W. E. Longstaff, H. Radjavi, *Spectral conditions and reducibility of operator semigroups*, Indiana Univ. Math. J. **41** (1992), no. 2, 449–464. MR1183353 (94a:47069)
15. W.A.J. Luxemburg, A.C. Zaanen, *Riesz spaces I*, North Holland, Amsterdam 1971. MR0511676 (58 #23483)
16. P. Meyer-Nieberg, *Banach lattices*, Springer-Verlag Berlin Heidelberg 1991. MR1128093 (93f:46025)
17. A.J. Michaels, *Hilden's simple proof of Lomonosov's invariant subspace theorem*, Adv. in Math. **25** (1977), 56–58. MR0500214 (58 #17893)
18. B. de Pagter, *Irreducible compact operators*, Math. Z. **192** (1986), 149–153. MR0835399 (87d:47052)
19. H. Radjavi, *On reducibility of semigroups of compact operators*, Indiana Univ. Math. J. **39** (1990), 499–515. MR1089051 (91m:47009)
20. H. Radjavi, P. Rosenthal, *From local to global triangularization*, J. Funct. Anal. **147** (1997), 443–456. MR1454489 (98j:47010)
21. C.J. Read, *Quasinilpotent operators and the invariant subspace problem*, J. London Math. Soc. (2) **56** (1997), 595–606. MR1610408 (98m:47004)
22. G.-C. Rota and W. G. Strang, *A note on the joint spectral radius*, Indag. Math. **22** (1960), 379–381. MR0147922 (26 #5434)
23. H.H. Schaefer, *Topologische Nilpotenz irreduzibler Operatoren*, Math. Z. **117** (1970), 135–140. MR0276802 (43 #2542)
24. H.H. Schaefer, *Banach lattices and positive operators*. (Grundlehren Math. Wiss. Bd. 215) Springer, Berlin Heidelberg New York 1974. MR0423039 (54 #11023)
25. V.S. Shulman, *On invariant subspaces of Volterra operators* (in Russian), Funk. Anal. i Prilozen. **18** (1984), 84–85. MR0745716 (85g:47008)
26. V.S. Shulman, Yu.V. Turovskii, *Joint spectral radius, operator semigroups and a problem of W. Wojtynski*, J. Funct. Anal., to appear. MR1795957 (2002d:47099)
27. Yu.V. Turovskii, *Volterra semigroups have invariant subspaces*, J. Funct. Anal. **162** (1999), 313–322. MR1682061 (2000d:47017)
28. Yu.V. Turovskii, *Spectral properties of certain Lie subalgebras and the spectral radius of subsets of a Banach algebra* (in Russian), Spectral theory of operators and its applications **6** (1985), 144–181, "Elm", Baku. MR0849356 (87k:46102)
29. A.C. Zaanen, *Riesz spaces II*, North Holland, Amsterdam 1983. MR0704021 (86b:46001)
30. Y. Zhong, *Irreducible semigroups of functionally positive nilpotent operators*, Trans. Amer. Math. Soc. **347** (1995), 3093–3100. MR1264835 (95j:47008)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR0087906 (19,434a) 46.1X
Dye, H. A.; **Phillips, R. S.**
Groups of positive operators.
Canad. J. Math. **8** (1956), 462–486.

Let $C_0(X)$ denote the space of all real-valued continuous functions $f(x)$ vanishing at infinity of the locally compact Hausdorff space X, metrized by the norm $\|f\| = \sup_x |f(x)|$, and let $\sigma \to U_\sigma$ be a representation of the group G of homeomorphisms of X by bounded positive operators on $C_0(X)$. Then U_σ admits a factorization $U_\sigma = L_{\theta(x,\sigma)} T_\sigma$ into the flow T_σ $(T_\sigma f(x) = f(x\sigma))$ and the multiplication L_θ $(L_{\theta(x,\sigma)} f(x) = \theta(x,\sigma) f(x))$, where, for each $\sigma, \theta(x,\sigma)$ is a positive continuous function on X bounded away from 0 and ∞ and satisfying $\theta(x,\sigma\tau) = \theta(x\sigma,\tau)\theta(x,\sigma)$, $\theta(x,e) = 1$ (e is the identity of G). The representation U_σ is equivalent to a pure flow ${T_\sigma}^{(1)}$ $(L_{\theta'} U_\sigma {L_{\theta'}}^{-1} = {T_\sigma}^{(1)})$ if and only if θ has the form $\theta(x,\sigma) = g(x)/g(x\sigma)$. Such is the case if the flow $x \to x\sigma$ is ergodic. The automorphism group of the group of all positive operators on $C_0(X)$ pertaining to a given flow is discussed, yielding the characterization of the group of flow-related automorphisms modulo inner automorphisms. In these discussions, a cohomology argument on $\theta(x,\sigma)$ is introduced as an algebraic vehicle. The canonical factorization $U_\sigma = L_\theta T_\sigma$ implies that the infinitesimal generator of a strongly continuous one-parameter group of bounded positive operators on $C_0(X)$ is, under appropriate conditions, represented as a differential operator of the first order. *K. Yosida*

MR0194893 (33 #3099) 47.10
Esajan, A. R.
On estimating the spectrum of the sum of positive semi-commuting operators. (Russian)
Sibirsk. Mat. Ž. **7** 1966 460–464

Let E be a Banach space with a positive cone. The positive operators $B_1, B_2, \cdots, B_s$ in E are said to be semi-commuting in case $B_i B_j \le B_j B_k$ $(i < j; i, j = 1, 2, \cdots, s)$. Conditions are given under which it can be concluded that the spectral radius satisfies $r(\sum_{k=1}^s B_k) \le \sum_{k=1}^s r(B_k)$. Two results are given which provide conditions assuring that if A and B are non-zero positive operators, then there does not exist a number α with $0 < \alpha < 1$ such that $AB \le \alpha BA$. *R. G. Bartle*

MR734664 (85i:47038) 47B55 47D10
Greiner, Günther (D-TBNG)
A spectral decomposition of strongly continuous groups of positive operators.
Quart. J. Math. Oxford Ser. (2) **35** (1984), *no. 137,* 37–47.

Let A be the generator of a strongly continuous group $(T(t))_{t\in\mathbf{R}}$ of positive operators on an (order complete) Banach lattice E. If there exists a $\mu\in\rho(A)\cap\mathbf{R}$ (hence the line $\operatorname{Re}\lambda=\mu$ belongs to $\rho(A)$) then E is the direct sum of $T(t)$-invariant projection bands $I_>$ and $I_<$ such that $\sigma(A|I_>)$ [resp. $\sigma(A|I_<)$] consists of all points $\lambda\in\sigma(A)$ with $\operatorname{Re}\lambda>\mu$ [resp. $\operatorname{Re}\lambda<\mu$]. The projection band $I_>$ [resp. $I_<$] is the set of all $x\in E$ with the property $(\mu-A)^{-1}|x|\le 0$ [resp. ≥ 0]. As a consequence, if $\mu\in\rho(A)\cap\mathbf{R}$ there exist positive constants m,M such that $\|T(t)x\|\le Me^{\mu t}x$ for $t\ge 0$, $x\in I_<$ and $\|T(t)x\|\ge me^{\mu t}x$ for $t\ge 0,\ x\in I_>$.

Heinz Langer

MR696523 (84f:47046) 47D10 22B99
Greiner, G.; **Groh, U.**

A Perron Frobenius theory for representations of locally compact abelian groups.

Math. Ann. **262** (1983), *no. 4*, 517–528.

Representations of a general locally compact abelian group G as groups of positive operators on a Banach lattice are studied with special reference to the properties of spectrum, point spectrum, and essential spectrum. It is shown in particular that the spectrum and the point spectrum are cyclic subsets of the dual group $\hat{G}$, and that they specialize to subgroups of $\hat{G}$ in the irreducible case. *Y. Domar*

MR981746 (90e:46037) 46H05 46B30 46H30
Grobler, J. J. [Grobler, Jacobus J.] (SA-POTCH)
The zero-two law in Banach lattice algebras.
Israel J. Math. **64** (1988), *no. 1,* 32–38.

This note improves on theorems of H. H. Schaefer [same journal **59** (1987), no. 2, 241–244; MR0920086 (88m:47066)] and A. R. Schep ["A remark on the uniform zero-two law for positive contractions", Preprint; per bibl.]. The present note proves that if A is a unital Banach lattice algebra and if a is an element of A^+ with $\|a\| \leq 1$, then either $\|a^{n+1} - a^n\| = 2$ for each nonnegative integer n, or else $\|a^{n+1} - a^n\| \to 0$ as $n \to \infty$; also, the peripheral spectrum of a is cyclic. *M. M. Day*

MR1354120 (96i:47001) 47A10 46H05
Grobler, J. J. [Grobler, Jacobus J.] (SA-POTCH); **Huijsmans, C. B.** (NL-LEID)
Doubly Abel bounded operators with single spectrum. (English summary)
Quaestiones Math. **18** (1995), *no. 4,* 397–406.

A classical result due to Gel′fand asserts that an element x of the unital Banach algebra A is the identity e if $\sigma(x)=\{1\}$ and x is doubly power-bounded. The latter means that $\|x^n\|\le C$ for all integers n. Here $\sigma(x)$ is the spectrum of x and C is a positive constant. There are several generalizations or variations. Recently, M. Mbekhta and J. Zemánek [C. R. Acad. Sci. Paris Sér. I Math. **317** (1993), no. 12, 1155–1158; MR1257230 (95b:47010)] proved the same conclusion as in Gel′fand's theorem but with doubly Cesàro bounded elements instead of doubly power-bounded elements. This means that one has $\|M_n(x)\|\le C$ and $\|M_n(x^{-1})\|\le C$, where $M_n(x)=(e+x+\cdots+x^n)/(n+1)$.

The main result of this paper extends the result of Mbekhta and Zemánek to doubly Abel bounded elements. This is obtained by replacing Cesàro means with Abel means. The relation between the two notions is also analyzed for Banach algebras and Banach lattice algebras. *Catalin Badea*

MR528730 (80b:15006) 15A09
Haynsworth, Emilie; **Wall, J. R.**
Group inverses of certain nonnegative matrices.
Linear Algebra Appl. **25** (1979), 271–288.

The group inverse of a matrix A, if it exists, is the unique matrix $A^{\#}$ which satisfies $AXA = A$, $XAX = X$, and $AX = XA$. Let $S = \{A^k\colon k$ a positive integer$\}$. The authors characterize (1) the nonnegative matrices A for which $A^{\#} \in S$, (2) the nonnegative matrices A for which the Moore-Penrose inverse $A^+ \in S$, and (3) the stochastic matrices A for which $A^{\#} \in S$. *Thomas H. Foregger*

MR629613 (82j:15004) 15A09 15A48
Haynsworth, Emilie; **Wall, J. R.**
Group inverses of certain positive operators.
Linear Algebra Appl. **40** (1981), 143–159.

If K is a cone in $\mathbf{R}^n$, then an $n \times n$ matrix A which maps K into K is called a positive operator on K. The set of all positive operators on K is denoted by $\pi(K)$. In a previous paper, the authors have characterized all nonnegative matrices with the property that the group inverse $(A^{\#})$ or the Moore-Penrose inverse (A^{+}) of the matrix is equal to some power of the matrix [same journal **25** (1979), 271–288; MR0528730 (80b:15006)]. The purpose of the paper under review is to generalize these results to certain positive operators on certain polyhedral cones. More precisely, the aforementioned $n \times n$ matrices A have the following form: There exist an $n \times r$ matrix P with $P^{\intercal} P = I_r$ and an $r \times r$ matrix $M \geq 0$ such that $AP = PM$ and $A \in \pi(K)$ for $K = G(P)$, the polyhedral cone generated by the columns of P. Necessary conditions are then given under which $A^{\#}$ or A^{+} is equal to A^k for some positive integer k.

J. J. Buoni

MR1969064 (2004b:15041) 15A48 47B60
Herzog, Gerd (D-KLRH-1)
Generalized M-matrices and ordered Banach algebras. (English summary)
Special issue on nonnegative matrices, M-matrices and their generalizations (Oberwolfach, 2000).
Linear Algebra Appl. **363** (2003), 125–131.

Summary: "A matrix $A \in \mathbb{R}^{n\times n}$ is an M-matrix if and only if the mapping $x \mapsto -Ax$ is quasimonotone increasing (qmi) and if the right spectral bound of $-A$ is negative. Here qmi is meant with respect to the natural cone $K = \{x \in \mathbb{R}^n\colon\ x_k \geq 0\}$. One possibility of generalizing M-matrices is to consider qmi linear mappings on $\mathbb{R}^n$ with respect to other cones $K \subseteq \mathbb{R}^n$. We present results on such mappings in the Banach algebra setting and discuss some special cones. Moreover, by means of one-sided estimates it is possible to get information on the right spectral bound of qmi mappings."

References

1. A. Berman, R.J. Plemmons, Cones and iterative methods for best least squares solutions of linear systems, SIAM J. Numer. Anal. 11 (1974) 145–154. MR0348984 (50 #1478)
2. T. Damm, D. Hinrichsen, Matrix (in)equalities for linear stochastic systems, in: Proceedings MTNS 1998, Padua.
3. G. Herzog, One-sided estimates for linear quasimonotone increasing operators, Numer. Funct. Anal. Opt. 19 (1998) 549–555. MR1636462 (99k:47077)
4. G. Herzog, On positive solutions of linear second order equations in Banach spaces, Demonstratio Math. 34 (2001) 587–591. MR1853736 (2002f:34131)
5. G. Herzog, R. Lemmert, On quasipositive elements in ordered Banach algebras, Stud. Math. 129 (1998) 59–65. MR1611855 (99g:46061)
6. G. Herzog, R. Lemmert, On ordered spaces of polynomials, Linear Algebra Appl. 320 (2000) 199–203. MR1795729 (2001h:47056)
7. G. Herzog, R. Lemmert, On Riccati equations in ordered Banach algebras, Demonstratio Math., in press. MR1941348 (2003j:34107)
8. M.A. Krasnosel'skii, G.M. Vainikko, P.P. Zabreiko, Ya.B. Rutitskii, V.Ya. Stetsenko, Approximate Solutions of Operator Equations, Wolters-Noordhoff Publ., Groningen, 1972. MR0385655 (52 #6515)
9. R. Lemmert, P. Volkmann, On the positivity of semigroups of operators, Comment. Math. Univ. Carolinae 39 (1998) 483–489. MR1666770 (99m:47048)
10. I. Marek, B. Szyld, Comparison theorems for weak splittings of bounded operators, Numer. Math. 58 (1990) 387–397. MR1077585 (92f:65070)
11. R.H. Martin, Nonlinear Operators and Differential Equations in Banach Spaces, Robert E. Krieger Publ., New York, 1987. MR0887947 (88i:34125)
12. H. Raubenheimer, S. Rode, Cones in Banach algebras, Indag. Mathem. 7 (1996) 489–502. MR1620116 (99i:46035)
13. W.T. Reid, Riccati Differential Equations, Academic Press, New York, 1972. MR0357936 (50 #10401)
14. H.H. Schaefer, Topological Vector Spaces, Springer Verlag, New York, 1999. MR0342978 (49 #7722)
15. H. Schneider, Positive operators and an inertia theorem, Numerische Math. 7 (1965) 11–17. MR0173678 (30 #3888)

16. H. Schneider, M. Vidyasagar, Cross-positive matrices, SIAM J. Numer. Anal. 7 (1970) 508–519. MR0277550 (43 #3283)
17. J. Schröder, Operator Inequalities, Academic Press, New York, 1980. MR0578001 (84k:65063)
18. R.J. Stern, Generalized M-matrices, Linear Algebra Appl. 41 (1981) 201–208. MR0649727 (83e:15023)
19. R.J. Stern, M. Tsatsomeros, Extended M-matrices and subtangentiality, Linear Algebra Appl. 97 (1987) 1–11. MR0916776 (88m:15005)
20. R.J. Stern, H. Wolkowicz, Exponential nonnegativity on the ice cream cone, SIAM J. Matrix Anal. Appl. 12 (1991) 160–165. MR1082333 (92j:15013)
21. A.N. Stokes, A special property of the matrix Riccati equation, Bull. Austral. Math. Soc. 10 (1974) 245–253. MR0342748 (49 #7493)
22. P. Volkmann, Gewöhnliche Differentialungleichungen mit quasimonoton wachsenden Funktionen in topologischen Vektorräumen, Math. Z. 127 (1972) 157–164. MR0308547 (46 #7661)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1611855 (99g:46061) 46H05
Herzog, Gerd (D-KLRH-1); **Lemmert, Roland** (D-KLRH-1)
On quasipositive elements in ordered Banach algebras. (English summary)
Studia Math. **129** (1998), *no. 1,* 59–65.

Summary: "Let a real Banach algebra A with unit be ordered by an algebra cone K. We study the elements $a \in A$ with $\exp(ta) \in K,\ t \geq 0$."

MR1941348 (2003j:34107) 34G20 47N20
Herzog, Gerd (D-KLRH-1); **Lemmert, Roland** (D-KLRH-1)
On Riccati equations in ordered Banach algebras. (English summary)
Demonstratio Math. **35** (2002), *no. 4,* 783–790.

Summary: "We consider Riccati differential equations in ordered Banach algebras $\mathcal{A}$, and prove invariance and comparison theorems for the case when the right-hand side of a Riccati equation is quasimonotone increasing on the set of quasipositive elements (which are the quasimonotone increasing linear mappings in the case that $\mathcal{A}$ is the operator algebra of an ordered Banach space)."

MR1991762 (2004e:46054) 46H05
Herzog, Gerd (D-KLRH-1); **Schmoeger, Christoph** (D-KLRH-1)
A note on a theorem of Raubenheimer and Rode. (English summary)
Proc. Amer. Math. Soc. **131** (2003), *no. 11,* 3507–3509 (*electronic*).

From the text: "Let $(A, \|\cdot\|)$ be a complex Banach algebra with unit e and $\|e\| = 1$. A set $W \subseteq A$ is called an algebra wedge if W is closed, $W + W \subseteq W$, $\lambda W \subseteq W$ ($\lambda \geq 0$), $W \cdot W \subseteq W$, and $e \in W$. As usual, by setting $a \leq b$ whenever $b - a \in W$ we obtain a reflexive and transitive relation on A, and call A ordered by W. In the sequel let $\sigma_A(a)$ and $r(a)$ denote the spectrum and the spectral radius of $a \in A$, respectively.

"H. Raubenheimer and S. Mouton [Indag. Math. (N.S.) **7** (1996), no. 4, 489–502; MR1620116 (99i:46035)] proved the following version of the Perron-Frobenius Theorem. Theorem 1. Let A be ordered by an algebra wedge W such that the spectral radius is increasing on W. Then $r(a) \in \sigma_A(a)$ for all $a \in W$.

"The purpose of this note is to prove the converse of this result: Theorem 2. Let $a \in A$ be such that $r(a) \in \sigma_A(a)$. Then there exists an algebra wedge W such that the spectral radius is increasing on W, and $a \in W$."

References

1. Berberian, S.K.: Lectures in functional analysis and operator theory. Graduate Texts in Math- ematics, Springer-Verlag, New York, 1974. MR0417727 (54 #5775)
2. Raubenheimer, H., Rode, S.: Cones in Banach algebras. Indag. Mathem. **7** (1996), 489–502. MR1620116 (99i:46035)
3. Rudin, W.: Functional analysis. Second edition. International Series in Pure and Applied Mathematics. McGraw-Hill, Inc., New York, 1991. MR1157815 (92k:46001)

Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2341945 (2008g:46071) 46H05 47A12 47B60
Herzog, Gerd (D-KLRH-IAN); **Schmoeger, Christoph** (D-KLRH-IAN)
An example on ordered Banach algebras. (English summary)
Proc. Amer. Math. Soc. **135** (2007), *no. 12,* 3949–3954 (*electronic*).

A given (non-ordered) Banach algebra is slightly extended to a certain ordered Banach algebra. This makes it possible to apply techniques on ordered Banach algebras to obtained results strictly in terms of the original algebra; for instance, concerning numerical ranges, exponentials and roots. *Jaroslav Zemánek*

References

1. Baillet, M.: Sur les 'el'ements hermitiens d'une alg'ebre de Banach et les dilatations de certains op'erateurs normaloides. C. R. Acad. Sci., Paris, Ser. **A 281**, 1039-1042 (1975). MR0390822 (52 #11645)
2. Bonsall, F.F.; Duncan, J.: Numerical ranges of operators on normed spaces and of elements of normed algebras. London: Cambridge University Press. II (1971). MR0288583 (44 #5779)
3. Herzog, G.; Kunstmann, P.C.: Ma jorization of $C0$-semigroups in ordered Banach spaces. Comment. Math. Univ. Carolin. **47**, 47-54 (2006). MR2223966 MR2223966 (2007g:47064)
4. Herzog, G.; Lemmert, R.: On quasipositive elements in ordered Banach algebras. Stud. Math. **129**, 59-65 (1998). MR1611855 (99g:46061)
5. Hilgert, J.; Neeb, K.-H.: Lie-Gruppen und Lie-Algebren. Braunschweig: Vieweg (1991).
6. Li, C.-K.; Rodman, L.; Spitkovsky, I.M.: On numerical ranges and roots. J. Math. Anal. Appl. **282**, 329-340 (2003). MR2000347 (2004g:47009)
7. Martin, R.H.: Nonlinear operators and differential equations in Banach spaces. Pure and Applied Mathematics. New York etc.: John Wiley&Sons XI (1976). MR0492671 (58 #11753)
8. Mazur, S.: "Uber konvexe Mengen in linearen normierten R"aumen. Stud. Math. **4**, 70-84 (1933).
9. Raubenheimer, H.; Rode, S.: Cones in Banach algebras. Indag. Math., New Ser. **7**, 489-502 (1996). MR1620116 (99i:46035)
10. Yosida, K.: Functional analysis. Berlin-G"ottingen-Heidelberg: Springer-Verlag. XI (1965). MR0500055 (58 #17765)

Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2803823 (2012m:47116) 47L07 46B40
Herzog, Gerd (D-KIT-IAN); **Schmoeger, Christoph** (D-KIT-IAN)
Suprema of chains of operators. (English summary)
Positivity **15** (2011), *no. 2,* 343–349.

Given a real or complex Banach space E, let $\mathcal{L}(E)$ denote the Banach algebra of all continuous linear operators on E endowed with the operator norm and let $\mathcal{E} \subseteq \mathcal{L}(E)$ be a real Banach space with respect to this norm. Assume that $\mathcal{E}$ is ordered by a cone $\mathcal{K} \subseteq \mathcal{E}$. In the paper under review, the authors introduce a property (which they call property (P)) for the cone $\mathcal{K}$ and prove that if this property holds, then each chain which is bounded from above has a supremum. Two examples of spaces of operators $\mathcal{E} \subseteq \mathcal{L}(E)$ which are ordered by cones with property (P) are discussed. In the first example the space E is a complex Hilbert space and $\mathcal{E}$ is the real Banach space of all linear and symmetric operators in $\mathcal{L}(E)$, endowed with the cone $\mathcal{K} = \{A \in \mathcal{E} : \langle Ax, x\rangle \geq 0,\ \forall x \in E\}$. The second example is a general result and is therefore considered to be a proposition which states that if a real Banach space E is ordered by a regular cone K (i.e. each increasing and order bounded sequence is convergent) that satisfies the condition $K - K = E$, then the set $\mathcal{K} = \{A \in \mathcal{L}(E) : A(K) \subseteq K\}$ of all monotone operators is a cone in $\mathcal{E} = \mathcal{L}(E)$ and $\mathcal{K}$ has property (P). *Jan Hendrik Fourie*

References

1. Andô, T.: On fundamental properties of a Banach space with a cone. Pacific J. Math. **12**, 1163–1169 (1962) MR0150572 (27 #568)
2. Bondarenko, V.A.: Integral inequalities for a Volterra equation in a Banach space with a cone. Mat. Zametki **9**, 151–160 (1971) MR0284835 (44 #2059)
3. Borwein, J.M., Lewis, A.S.: Convex analysis and nonlinear optimization. Theory and examples. CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC, vol. 3. Springer, New York (2000) MR1757448 (2001h:49001)
4. Erné, M.: $\mathcal{W}$-Completeness and fixpoint properties. Arch. Math. (Brno) **24**, 147–155 (1988) MR0983232 (90e:06004)
5. Heinz, E.: Beiträge zur Störungstheorie der Spektralzerlegung. Math. Ann. **123**, 415–438 (1951) MR0044747 (13,471f)
6. Herzog, G.: An intermediate value theorem in ordered Banach spaces. Ann. Polon. Math. **98**, 63–69 (2010) MR2607486 (2011a:47102)
7. Krasnosels'kij, M.A.: Pravil'nye i vpolne pravil'nye konusy. Doklady Akad. Nauk. SSSR **135**, 255–257 (1960)
8. Lemmert, R., Redheffer, R.M., Volkmann, P.: Ein Existenzsatz für gewöhnliche Differentialgleichungen in geordneten Banachräumen. General inequalities, 5 (Oberwolfach, 1986), 381–390, Internat. Schriftenreihe Numer. Math., 80, Birkhäuser, Basel (1987) MR1018161 (90g:34064)
9. Lemmert, R.: Existenzsätze für gewöhnliche Differentialgleichungen in geordneten Banachräumen. Funkc. Ekvacioj, Ser. Int. **32**, 243–249 (1989) MR1019432 (90i:34096)
10. Markowsky, G.: Chain-complete posets and directed sets with applications. Algebra Universalis **6**, 53–68 (1976) MR0398913 (53 #2764)
11. Riesz, F., Sz.-Nagy, B.: Functional analysis. London-Glasgow: Blackie & Son, Ltd. XII

12. Schmidt, S.: Existenzsätze für gewöhnliche Differentialgleichungen in Banachräumen. Funkc. Ekvacioj, Ser. Int. **35**, 199–222 (1992) MR1189893 (94f:34121)
13. Uhl, R.: Smallest and greatest fixed points of quasimonotone increasing mappings. Math. Nachr. **248**–**249**, 204–210 (2003) MR1950728 (2003j:47079)
14. Volkmann, P.: Über Kegel, welche einer Bedingung von Lemmert genügen. Series Math. Cato. Debrec. No. 7. http://www.math.us.edu.pl/smdk/ (2000)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR886000 (88i:46010) 46A40 06F20 06F25
Huijsmans, C. B. (NL-LEID)
An inequality in complex Riesz algebras.
Studia Sci. Math. Hungar. **20** (1985), *no. 1-4,* 29–32.

Let L be a Riesz algebra (lattice ordered algebra) which is Archimedean and relatively uniformly complete. The author considers the vector space complexification $L+iL$ of L with modulus defined by $|\varphi| = |f+ig| = \sup\{|f\cos\theta + g\sin\theta|\colon\ 0 \le \theta \le 2\pi]$. The main result of this article is the fact that the triangle inequality $|\varphi\psi| \le |\varphi|\,|\psi|$ is valid for all φ, ψ in $L+iL$. *Rosalind Reichard*

MR934473 (89f:46106) 46J05 46B30 46H05
Huijsmans, C. B. (NL-LEID)
Elements with unit spectrum in a Banach lattice algebra.
Nederl. Akad. Wetensch. Indag. Math. **50** (1988), *no. 1,* 43–51.

Let E be a Banach lattice algebra with positive unit element. If a is positive and invertible with a^{-1} positive and the spectrum of a is $\{1\}$, then a is equal to the unit element. An elementary proof for this result is presented. It is elementary in the sense that it does not rely on representation theory for Banach lattices. The result is also a consequence of the work of H. H. Schaefer, M. P. H. Wolff and W. Arendt [Math. Z. **164** (1978), no. 2, 115–123; MR0517148 (80b:47048)] who used representation theory in their proof. *W. A. Feldman*

MR1340473 (96g:46013) 46B42 06F25 47B60
Huijsmans, C. B. (NL-LEID)
A characterization of complex lattice homomorphisms on Banach lattice algebras. (English summary)
First International Conference in Abstract Algebra (Kruger Park, 1993).
Quaestiones Math. **18** (1995), *no. 1-3,* 131–140.

In this paper, the author presents an interesting version of the celebrated theorem of W. Żelazko (which states that a bounded linear functional φ of a complex Banach algebra A with unit e is multiplicative (i.e. an algebraic homomorphism) if and only if, for all $a \in A$, $\varphi(a) \in \sigma$), for the case of complex Banach lattice algebras with a positive unit element e. For such algebras it is shown that the order ideal generated by e is a projection band and that its linear lattice homorphisms are multiplicative. It is then shown that a bounded linear functional φ is a lattice homorphism (and hence multiplicative) if, for every element a, $\varphi(a)$ is contained in the spectrum of the element which is the projection of a on the projection band generated by e.

{For the entire collection see MR1340467 (96c:00020)}

W. A. J. Luxemburg

MR826666 (87f:47055) 47B55 46A40
Huijsmans, C. B. (NL-LEID); **de Pagter, B.** **[de Pagter, Ben]** (NL-DELF)
Averaging operators and positive contractive projections.
J. Math. Anal. Appl. **113** (1986), *no. 1,* 163–184.

The authors are mainly concerned with generalizations of two theorems about positive contractive projections on $C_0(X)$. The generalizations are: (i) If A is an Archimedean semiprime f-algebra satisfying the Stone condition, then every positive contractive projection T on A satisfies the identity $T(a\cdot Tb)=T(Ta\cdot Tb)$ for $a,b\in A$. (ii) In addition, T is averaging if and only if the range of T is a subalgebra. (We recall that averaging operators are defined by the condition $T(a\cdot Tb)=Ta\cdot Tb$.) For $C_0(X)$, (i) is due to G. L. Seever [Pacific J. Math. **17** (1966), 159–166; MR0192356 (33 #581)] and (ii) to J. L. Kelley [Illinois J. Math. **2** (1958), 214–223; MR0103409 (21 #2179)].

In contrast to the analytic proofs given by Seever and Kelley, the authors' proofs are algebraic and order-theoretic. The analysis is neatly packaged in results taken from the theory of orthomorphisms on vector lattices. Some very elegant techniques are used. The Schwarz inequality is extended to Archimedean semiprime f-algebras. The proof of this is standard once one has proved the "standard" fact that $\lambda^2a+2\lambda b+c\geq 0$ for all real λ and all a,b,c in an Archimedean semiprime f-algebra, implies that $b^2\leq ac$. The Schwarz inequality is important in subsequent arguments and can also be used to prove Proposition 4.6: $T(a\cdot Te)=Ta$ $(a\in A)$, when A is an Archimedean f-algebra with unit element e and T a positive contractive projection on A. The range of a positive projection on an Archimedean f-algebra is known to be a vector lattice, in its induced order, under $a\overset{*}{\vee}b=T(a\vee b)$. It is also an f-algebra under the new multiplication $a*b=T(ab)$.

The paper is well written, has an excellent bibliography, and contains the key counterexamples to show that its hypotheses are necessary as well as sufficient.

S. J. Bernau

MR566945 (83a:90182) 90C48 49B27
Itskovich, I. A.
Linear models of optimization in real Banach algebras admitting of natural ordering. (Russian)
Models and methods of investigating economic systems (Russian), pp. 60–90, *"Nauka" Sibirsk. Otdel., Novosibirsk*, 1979.

The subject of this paper is linear programming in real commutation Banach algebras with an identity. In Section 1 it is shown that such an algebra E can be ordered if it satisfies the following conditions. (A) E has nontrivial ideals. (B) The intersection of all its maximal ideals consists of only the zero element. (C) The dimension of its quotient space relative to any of its maximal ideals is 1. This means that a pointed convex closed solid nonnegative cone $K \subset E$ exists such that (a) the identity $e \in K$, (b) $x \in K$ and $y \in K \Rightarrow xy \in K$ and (c) $\operatorname{int} K$ is a group under multiplication.

In Section 2, the set F of all multiplicative functionals over E is considered together with continuous linear functions determined by algebra elements according to the rule: $x(f) := (x, f)$, $x \in E$, $f \in F$.

In Section 3, the operation of multiplication of linear functionals over E by elements of E is introduced: for any $a \in E$ and $g \in E^*$, ag is a functional from E^* such that $(x, ag) = (ax, g)$ for $x \in E$.

In Section 4, it is demonstrated that the exponential function $\exp(x) := \sum_{n=0}^{\infty} x^n/n!$, $x \in E$, maps E onto the set $\operatorname{int} K$ of all positive elements.

Let E_1 and E_2 be two algebras satisfying conditions (A)-(C), K_1 and K_2 be their nonnegative cones and A be a linear bounded operator from E_1 into E_2. Section 5 deals with the linear programming problem $\lambda_0 := \sup\{(x, c) | Ax + y = b, x \in K_1, y \in K_2\}$, $b \in E_2$, $c \in E_1^*$, and its dual $\mu_0 := \inf\{(b, u) | A^*u - v = c, u \in K_2^*, v \in K_1^*\}$. Here, as usual, $K^* := \{v \in E^* \colon (x, v) \geq 0 \text{ for all } x \in K\}$ is the conjugate cone for K. In particular, the duality theorem ($\lambda_0 = \mu_0$) and the criteria of optimality (feasible elements x, y and u, v of primal and dual problems respectively are optimal $\Leftrightarrow$ they are connected by the complementary slackness conditions $xv = 0$ and $yu = 0$) are proved.

Section 6 is devoted to the question of variables scaling in primal and dual problems.

In Section 7 the self-dual problem (∗) $\inf\{(b, f) \colon Af \leq b, f \geq 0\}$, $f \in E^*$, $b \in E$, with a skew-symmetric operator A (i.e. $A^* = -A$) is examined. It is proved that this infimum is zero and a feasible element f is optimal $\Leftrightarrow (b - Af)f = 0$ (complementary slackness condition). An example of such a problem, namely

$$\inf\left\{\int_0^1 b(t)\,df(t) | df(t) \geq 0, \int_0^1 (s-t)\,df(t) \leq b(s)\right\},$$

$b \in C[0,1]$, is investigated in full in Section 8.

In concluding Section 9 a pair of dual LP problems

$$(1) \qquad \sup\{(b_2, f_2) \colon Df_2 + y_1 = b_1, f_2 \in K_2^*, y_1 \in K_1\},$$

$$(2) \qquad \inf\{(b_1, f_1) \colon D^*f_1 - y_2 = b_2, f_1 \in K_1^*, y_2 \in K_2\}$$

is considered, where $D \colon E_2^* \to E_1$ is a linear bounded operator and D^* is its adjoint. It is stressed that this pair is equivalent to the problem $\inf\{(b, f) | Af + y = b, f \in K^*, y \in K\}$ of type (∗), where $b := [b_1, -b_2]$, $y := [y_1, y_2]$, $f := [f_1, f_2]$, $K := K_1 \times K_2$ and the operator $A \colon E_1^* \times E_2^* \to E_1 \times E_2$, $Af := [Df_2, -D^*f_1]$, is skew-symmetric. It is shown that (i) the cone $B_1 := D(K_2^*) + K_1$ coincides with $E_1 \Leftrightarrow \exists f_2 \in K_2^* \setminus 0$ with $Df_2 < 0$

(something similar is valid for $B_2 := D^*(K_1^*) - K_2$); (ii) if $B_i = E_i$, then the feasible set in problem (i) is not bounded for every $b_i \in E_i$, $i = 1, 2$.

{For the entire collection see MR0566942 (81a:90006)}

Z. Waksman

MR2465374 (2009k:46047) 46E05 46J10 47B33 47B38
Jiménez-Vargas, A. (E-ALM-AMA); **Villegas-Vallecillos, Moisés** (E-ALM-AMA)
Order isomorphisms of little Lipschitz algebras. (English summary)
Houston J. Math. **34** (2008), *no. 4,* 1185–1195.

Let (X,d) be a metric space and $\alpha\in(0,1]$. A scalar-valued function is said to satisfy the Lipschitz condition with respect to the metric d^α defined by $d^\alpha(x,y)=(d(x,y))^\alpha$ if there exists a positive constant k such that

$$|f(x)-f(y)|\le k\cdot d^\alpha(x,y),\quad \text{for all } x,y\in X.$$

Such a function is called a Lipschitz-α function and the set of bounded Lipschitz-α functions forms a Banach space under the norm $\|f\|_\alpha=p_\alpha(f)+\|f\|_\infty$, where

$$p_\alpha(f)=\sup\{|f(x)-f(y)|/d^\alpha(x,y)\colon\ x,y\in X,\ x\ne y\}$$

and $\|f\|_\infty$ is the usual sup norm. This Banach space is denoted by $\mathrm{Lip}(X,d^\alpha)$. (It should be noted that sometimes one considers the norm on $\mathrm{Lip}(X,d^\alpha)$ given by $\max(p_\alpha(f),\|f\|_\infty)$. However, the sum norm given in the definition above satisfies the multiplicativity property needed for a Banach algebra.) By $\mathrm{lip}(X,d^\alpha)$ (called a little Lipschitz algebra) is meant the closed subspace of $\mathrm{Lip}(X,d^\alpha)$ consisting of all those Lipschitz-α functions with the property that for each $\varepsilon>0$, there exists $\delta>0$ such that $0<d(x,y)<\delta$ implies $|f(x)-f(y)|/d^\alpha(x,y)<\varepsilon$. We note that both $\mathrm{Lip}(x,d^\alpha)$ and $\mathrm{lip}(X,d^\alpha)$ are unital self-adjoint commutative Banach algebras with respect to pointwise multiplication and they are ordered vector spaces as well, where $f\ge 0$ means that $f(x)\ge 0$ for all $x\in X$. A linear map T between two ordered vector spaces of scalar-valued functions is called an order isomorphism if it is bijective and both T, T^{-1} are order preserving.

Order isomorphisms between $\mathrm{Lip}(X,d)$-spaces have been studied, for example, by N. Weaver [Pacific J. Math. **164** (1994), no. 1, 179–193; MR1267506 (95b:46031)] and M. I. Garrido Carballo and J. Á. Jaramillo [Monatsh. Math. **141** (2004), no. 2, 127–146; MR2037989 (2004k:46034)], who obtained results in the spirit of the Banach-Stone theorem for spaces of continuous functions. The goal of the paper under review is to determine the form of all order isomorphisms between little Lipschitz algebras. Here is the main theorem:

Let (X,d_X) and (Y,d_Y) be compact metric spaces and let $\alpha,\beta\in(0,1)$. A bijective linear map $T\colon\mathrm{lip}(X,d_X^\alpha)\to\mathrm{lip}(Y,d_Y^\beta)$ is an order isomorphism if and only if there exist a nonvanishing positive function a in $\mathrm{lip}(Y,d_Y^\beta)$ and a Lipschitz homeomorphism h from (Y,d_Y^β) onto (X,d_X^α) such that T is of the form

$$T(f)=a\cdot(f\circ h),\quad \text{for all } f\in\mathrm{lip}(X,d_X^\alpha).$$

Moreover,

$$T^{-1}(g)=[1/(a\circ h^{-1})]\cdot(g\circ h^{-1}),\quad \text{for all } g\in\mathrm{lip}(Y,d_Y^\beta).$$

The function h is obtained by showing that the support of the functional $\delta_y\circ T$ is a singleton $x\in X$, and by defining $h(y)=x$. Here δ_y, for $y\in Y$, means the evaluation functional, and the support of $\delta_y\circ T$ is the set of all x such that for each neighborhood U of x there is a Lipschitz-α function f whose co-zero set is contained in U and such that $(\delta_y\circ T)(f)\ne 0$. A consequence of the theorem is that the two little Lipschitz algebras are order isomorphic if and only if the corresponding metric spaces are Lipschitz homeomorphic.

Richard Fleming

References

1. Y. A. Abramovich, *Multiplicative representation of disjointness preserving operators,* Indag. Math. **45** (1983), 265–279. MR0718068 (85f:47040)
2. W. Arendt and J. De Canniéere, *Order isomorphisms of Fourier algebras,* J. Funct. Anal. **50** (1983), 1–7. MR0689996 (84h:43011)
3. M. I. Garrido and J. A. Jaramillo, *Homomorphisms on function lattices,* Monatsh. Math. **141** (2004), 127–146. MR2037989 (2004k:46034)
4. M. S. Monfared, *Extensions and isomorphisms for the generalized Fourier algebras of a locally compact group,* J. Funct. Anal. **198** (2003), 413–444. MR1964545 (2004d:46058)
5. B. Pavlović, *Discontinuous maps from Lipschitz algebras,* J. Funct. Anal. **155** (1998), 436–454. MR1624565 (99d:46073)
6. D. Sherbert, *Banach algebras of Lipschitz functions,* Pacific J. Math. **13** (1963), 1387–1399. MR0156214 (27 #6143)
7. D. Sherbert, *The structure of ideals and point derivations in Banach algebras of Lipschitz functions,* Trans. Amer. Math. Soc. **111** (1964), 240–272. MR0161177 (28 #4385)
8. N. Weaver, *Lattices of Lipschitz functions,* Pacific. J. Math. **164** (1994), 179–193. MR1267506 (95b:46031)
9. N. Weaver, *Nonatomic Lipschitz spaces,* Studia Math. **115** (1995), 277–289. MR1351242 (96k:46032)
10. N. Weaver, *Order completeness in Lipschitz algebras,* J. Funct. Anal. **130** (1995), 118–130. MR1331979 (96f:46048)
11. N. Weaver, *Lipschitz Algebras,* World Scientific, Singapore, 1999. MR1832645 (2002g:46002)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR0044040 (13,360b) 46.3X
Kadison, Richard V.
A representation theory for commutative topological algebra.
Mem. Amer. Math. Soc., **1951,** (1951). *no. 7, 39 pp.*

This paper is devoted to the study of the functional representation method and the unification of theorems developed in recent years after the early work of Stone, Gelfand, Kakutani and others on the commutative spectral theory. They center around the characterization of the space $C(X)$ of all real or complex-valued continuous functions on a compact X and rest on the availability of the Tychonoff, Stone-Weierstrass and Krein-Milman theorems. The author starts out (§2) with the following general result. Theorem 2.1: Let V be an Archimedian ordered vector space with an order unit. Then the natural map of V into the real $C(X)$, where X is the compact weak closure of the set of all extreme maximal ideals of V, is an isomorphism with respect to the units. Here the representation space is not the too big set of all maximal ideals, so the result applies at once to the next cases. In §3 the Stone algebra theorem is derived from the preceding. Theorem 3.1: Let A be a real algebra which is an Archimedian ordered vector space with unit (in both senses), complete in the natural norm, where $x \geq 0$, $y \geq 0$ imply $xy \geq 0$. Then A is isomorphic to a real $C(X)$. The algebra A is not assumed commutative or associative nor are squares required to be positive; these facts follow from the others. §4 derives the Kakutani-Krein lattice theorem from Theorem 2.1 by showing that the natural map is now onto and that the set of extreme maximal ideals is already closed. §5 treats the Banach space characterization of a real $C(X)$ due to Arens and Kelley by methods emphasizing the order situation. The long §5 is devoted to proving some real Banach algebra representation theorems by purely real methods and to passing from these to the basic real and complex theorems of Gelfand. In doing so the analytic functions techniques are not avoided although a version of the real Mazur-Gelfand theorem is proved by non-complex methods. Theorem 6.3: Let A' be a real Banach algebra ($\|xy\| \leq \|x\| \cdot \|y\|$) containing a dense subalgebra A with unit 1, $\|1\| = 1$, such that $(x^2+1)^{-1}$ exists in A' for $x \in A$ and $(x^2-1)^{-1}$ does not exist in A' for $x \in A$, $\|x\| = 1$. Then A' is isomorphic to a real $C(X)$. The author points out that the theorem as it stands with conditions on the incomplete A is needed later on. Theorem 6.3 is used to show that a real Banach algebra with unit 1, $\|1\| = 1$, where $\|x^2\| = \|x\|^2$ and $(x^2+1)^{-1}$ always exists is isomorphic to a real $C(X)$ and to get the important representation as a complex $C(X)$ of a commutative B^*-algebra (complex Banach algebra with map $x \to x^*$ such that $(x+y)^* = x^* + y^*$, $(\lambda x)^* = \overline{\lambda} x^*$, $x^{**} = x$, $(xy)^* = y^* x^*$, $\|xx^*\| = \|x\| \cdot \|x^*\|$) with unit. §7 reduces any polynomial identity on the norm to that of Gelfand. Finally, §8 applies these results to commutative C^*-algebras (uniformly closed self-adjoint algebras) of operators in a Hilbert space. Most of the proofs in this paper are neater than those already existing in the fairly complete list of references at the end.

L. Nachbin

MR0054175 (14,883e) 46.3X
Kelley, J. L.; **Vaught, R. L.**
The positive cone in Banach algebras.
Trans. Amer. Math. Soc. **74,** (1953). 44–55

Lemmas are proved on ordered Banach spaces R such that the non-negative cone C contains an element e where $\|e\| = 1$ and $\{y| \, \|e - y\| \leq 1\} \subset C$. For example, in such a space (a) $\|f\| = f(e)$ for $f \in C'$, the cone polar to C, and (b) dist $(-x, C) = \sup \{f(x)|f \in C'$ and $\|f\| \leq 1\}$. It is then observed that if a Banach algebra is ordered via a positive cone which is the closure of the set of sums of squares (sums of elements xx^* when ordering self-adjoint elements of *-algebras), the hypotheses of these lemmas apply. By this means in a systematic manner a number of known and new theorems are proved. In the latter category is the following: let $f(t) = \sum_{n=-\infty}^{\infty} a_n e^{int}$ be absolutely convergent and realvalued and let S be the class of all (two-ended sequences) which are zero except for a finite number of terms with non-negative indices; then

$$\max \left[\sup \left\{ f(t) \middle| 0 \leq t < 2\pi \right\}, 0 \right] = \inf \left\{ \sum_{n=-\infty}^{\infty} |a_n + \sum_{m=-\infty}^{\infty} b_m \bar{b}_{m-n}| \middle| b \in S \right\}.$$

S. Sherman

References

1. Richard Arens, *Representations of ∗-algebras*, Duke Math. J. vol. 14 (1947) pp. 269-282. MR0021250 (9,44b)
2. I. Gelfand and M. Naimark, *On the imbedding of normed rings into the ring of operators in Hilbert space*, Rec. Math. (Mat. Sbornik) N.S. vol. 12 (1943) pp. 197-213. MR0009426 (5,147d)
3. I. Gelfand and M. Naimark, *Normed rings with involutions and their representations*, Izvestiya Akademii Nauk SSSR. Ser. Mat. vol. 12 (1948) pp. 445-480. MR0026763 (10,199a)
4. P. R. Halmos, *Measure theory*, New York, 1950. MR0033869 (11,504d)
5. R. V. Kadison, *A representation theory for commutative topological algebra*, Memoirs of the American Mathematical Society, no. 7, 1951. MR0044040 (13,360b)
6. I. Kaplansky, *Normed algebras*, Duke Math. J. vol. 16 (1949) pp. 399-418. MR0031193 (11,115d)
7. M. Krein and D. Milman, *On extreme points of regular convex sets*, Studia Mathematica vol. 9 (1940) pp. 133-137. MR0004990 (3,90a)
8. M. Krein and M. Rutman, *Linear operators leaving invariant a cone in a Banach space*, Uspehi Matematičeskih Nauk. vol. 3 (23) (1948) pp. 3-95. MR0027128 (10,256c)
9. D. A. Raikov, *To the theory of normed rings with involutions*, C. R. (Doklady) Acad. Sci. URSS. vol. 54 (1946) pp. 387-390. MR0019845 (8,469d)
10. I. E. Segal, *The group algebra of a locally compact group*, Trans. Amer. Math. Soc. vol. 61 (1947) pp. 69-105. MR0019617 (8,438c)
11. I. E. Segal, *Irreducible representations of operator algebras*, Bull. Amer. Math. Soc. vol. 53 (1947) pp. 73-88. MR0020217 (8,520b)

12. I. E. Segal, *Postulates for general quantum mechanics*, Ann. of Math. vol. 48 (1947) pp. 930-948. MR0022652 (9,241b)
13. S. Sherman, *Non-negative observables are squares*, Proceedings of the American Mathematical Society vol. 2 (1951) pp. 31-33. MR0039925 (12,617e)
14. M. H. Stone, *Applications of the theory of Boolean rings to general topology*, Trans. Amer. Math. Soc. vol. 41 (1937) pp. 375-481. MR1501905
15. G. Szegö, *Orthogonal polynomials*, Amer. Math. Soc. Colloquium Publications, vol. 23, New York, 1939.
16. A. Weil, *L'intégration dans les groupes topologiques et ses applications*, Actualités Scientifiques et Industrielles, no. 869, Paris, 1940.

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR0175937 (31 #213) 13.98 46.55
Krivine, J.-L.
Anneaux préordonnés. **(French)**
J. Analyse Math. **12** 1964 307–326

A sample of the ideas. A is a commutative ring with 1 over $\mathbf{Q}$ in which -1 is not a sum of squares. A preorder Π on A is a subset, closed under addition and multiplication, which contains all squares, but does not contain -1. Then $|\Pi| = \Pi \cap -\Pi$ is a proper ideal. An order is a preorder with $|\Pi| = 0$, and A is a pseudofield if Π is total, $|\Pi| \cup -|\Pi| = A$, and also $x \neq 0$ implies $xy \geq 1$ for some y. Then $A/|\Pi|$ is a pseudofield if and only if Π is maximal. Theorem: If Π is a preorder on A, then $x_1, \cdots, x_n$ do not all belong to $|\Pi'|$ for some maximal extension Π' of Π if and only if $\sum \lambda_i x_i \geq 1$ for some λ_i. Theorem: Let K be an ordered field, L a real closed extension. Then $p_1, \cdots, p_h$, elements of $K[x_1, \cdots, x_n]$, have no common root in L if and only if $\sum q_i p_i = 1 + \lambda_1 {r_1}^2 + \cdots + \lambda_t {r_t}^2$ for some q_i, r_j in $K[x_1, \cdots, x_n]$ and λ_j in K. Theorem: If A is a formally real field and $x, y \geq 0$ in no total order on A, then $\omega_0 + \omega_1 x + \omega_2 y + \omega_3 xy = -1$ for some sums of squares ω_i; an analogous extension of Artin's theorem on positive functions is presented.

The envelope E of Π is the intersection of all maximal Π' containing Π, and the radical R, that of all $|\Pi'|$. Elementary characterizations of E and R are given. If Π is Archimedean in the sense that each $x \leq n$ for some integer n, then E consists of all $x \geq -1/n$ (all $n > 0$), and R of all x such that $-1/n \leq x \leq 1/n$ (all $n > 0$). An Archimedean pseudofield is a subring of $\mathbf{R}$ containing $\mathbf{Q}$. Thus if Π is an Archimedean preorder, the maximal extensions Π' of Π correspond to homomorphisms χ from A into $\mathbf{R}$. The spectrum $\mathrm{Sp}(A)$, consisting of all such χ, is compact in a natural topology, and A is dense in the ring of continuous real functions on $\mathrm{Sp}(A)$. If A is an algebra over $\mathbf{R}$, the non-negative linear forms on A are precisely the forms $T(x) = \int \hat{x}\, d\mu$ for μ a positive Radon measure on $\mathrm{Sp}(A)$.

Applications to real Banach algebras are given. If x is positive on $\mathrm{Sp}(A)$, then $x = \alpha^2 + {x_1}^2 + \cdots + {x_n}^2$, $\alpha \neq 0$ in $\mathbf{R}$ and the x_i in A. The norm $\|x\|$ is the limit inferior of real $\lambda \geq 0$ such that both $\lambda + x$ and $\lambda - x$ are sums of squares. If A is $\mathcal{L}^1(N)$, N the natural numbers, as convolution algebra, then $\mathrm{Sp}(A) = [-1, 1]$, and one has a result on positive entire series on this interval. If A is $\mathcal{L}^1[0, \infty)$, with 1 adjoined, as convolution algebra, then $k \in \mathcal{L}^\infty$ satisfies $\iint k(x+y) f(x) f(y)\, dxdy \geq 0$ for all f in A if and only if $k(x) = \int e^{-px}\, d\mu(p)$ for some bounded positive μ. Application to commutative C^*-algebras is also included.

R. C. Lyndon

MR1825056 (2002k:43003) 43A35 43A10
Lashkarizadeh Bami, M. (IR-TPM)
Generalized positive definite functions and completely monotone functions on foundation semigroups. (English summary)
J. Sci. Islam. Repub. Iran **11** (2000), *no. 3,* 245–252.

The aim of this paper is to introduce and study completely monotone functionals on an ordered Banach algebra B into a proper H^*-algebra A, giving an integral representation for such functionals and to apply this theory to completely continuous monotone functions on weighted foundation semigroups. The main result is an integral representation for the generalized w-bounded continuous completely monotone A-valued functions with respect to positive A-valued measures on the space of w-bounded continuous nonnegative semicharacters on a foundation semigroup S with a Borel measurable weight function w.

The paper also contains a section devoted to generalized representations and positive-definite functions on weighted foundation semigroups in which a generalized version of Bochner's theorem on foundation semigroups is given. *Liliana Pavel*

MR2108364 (2005m:47086) 47D03 47A15 47B38 47B65
MacDonald, Gordon [**MacDonald, Gordon Wilson**] (3-PRIN-MS);
Radjavi, Heydar (3-DLHS-MS)
Standard triangularization of semigroups of non-negative operators. (English summary)
J. Funct. Anal. **219** (2005), *no. 1,* 161–176.

In the first part of the paper the authors consider the question when a (multiplicative) semigroup of non-negative (= positive) operators on $L^p(X,\mu)$ (for $1 \le p < \infty$) of the form scalar plus compact is triangularizable via standard subspaces if each operator in the semigroup is triangularizable via standard subspaces. In the terminology of Banach lattices, standard subspaces are precisely closed ideals (in fact, bands) of $L^p(X,\mu)$. The obtained results extend in a number of directions the known theorem that a semigroup of positive compact quasinilpotent operators on $L^p(X,\mu)$ has a nontrivial invariant band. This theorem was given simultaneously in the monograph of H. Radjavi and P. Rosenthal [*Simultaneous triangularization*, Springer, New York, 2000; MR1736065 (2001e:47001)] and, for the case of general Banach lattices, in the paper by the reviewer [Integral Equations Operator Theory **39** (2001), no. 3, 253–266; MR1818060 (2001m:47012)].

The second part of the paper introduces a generalized determinant function on positive operators which are of the form identity plus trace class. It is shown that the submultiplicativity of this generalized determinant function on a semigroup is equivalent to the standard triangularizability of the semigroup. This result generalizes the known finite-dimensional theorem (see Theorem 5.1.6 in the monograph mentioned above).

Roman Drnovšek

References

1. M.D. Choi, E.A. Nordgren, H. Radjavi, P. Rosenthal, Y. Zhong, Triangularizing semigroups of quasinilpotent operators with non-negative entries, Indiana Univ. Math. J. 42 (1993) 15–25. MR1218704 (94e:47009)
2. K. Davidson, Nest Algebras, in: Pitman Research Notes, Vol. 191, Longman, New York, 1988. MR0972978 (90f:47062)
3. B. de Pagter, Irreducible compact operators, Math. Z. 192 (1986) 149–153. MR0835399 (87d:47052)
4. R. Drnovšek, Common invariant subspaces for collections of operators, J. Integral Equations Operator Theory 39 (2001) 253–266. MR1818060 (2001m:47012)
5. R. Drnovšek, D. Kokol-Bukovšek, L. Livshits, G. MacDonald, M. Omladič, H. Radjavi, An irreducible semigroup of non-negative square-zero operators, J. Integral Equations Operator Theory 42 (2002) 449–460. MR1885443 (2003j:47051)
6. P. Halmos, V. Sunder, Bounded Integral Operators on L^2 Spaces, Springer, Berlin, 1978. MR0517709 (80g:47036)
7. A. Katavolos, H. Radjavi, Simultaneous triangularization of operators on a Banach space, J. London Math. Soc. 41 (2) (1990) 547–554. MR1072047 (91i:47058)
8. H. König, Eigenvalue distribution of compact operators, in: Operator Theory: Advances and Applications, Vol. 13, Birkhauser, Basel, 1986. MR0889455 (88j:47021)
9. A. Pietsch, Operator Ideals, North-Holland, Amsterdam, 1980. MR0582655 (81j:47001)
10. H. Radjavi, P. Rosenthal, From local to global triangularization, J. Funct. Anal. 147 (1997) 443–456. MR1454489 (98j:47010)

11. H. Radjavi, P. Rosenthal, Simultaneous Triangularization, Universitext, Springer, New York, 2000. MR1736065 (2001e:47001)
12. J. Ringrose, Superdiagonal forms for compact linear operators, Proc. London Math. Soc. 12 (3) (1962) 367–384. MR0136998 (25 #458)
13. G. Skandalis, Une notion de nucléarité en K-théorie, K-Theory (1) (1998) 549–574. MR0953916 (90b:46131)
14. D. Voiculescu, A non-commutative Weyl-von Neumann theorem, Rev. Roumame Math. Pures Appl. 21 (1976) 97–113. MR0415338 (54 #3427)
15. S. Wasserman, Exact C^*-algebras and Related Topics, RIMS Lecture Notes Series, vol. 19, 1994. MR1271145 (95b:46081)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR607012 (82c:46027) 46B30 46J10
Martignon, Laura
Banach f-algebras and Banach lattice algebras with unit.
Bol. Soc. Brasil. Mat. **11** (1980), *no. 1,* 11–17.

The author permutes some standard facts about representation theory for Archimedean lattice groups and Archimedean lattice rings, to produce a necessary and sufficient condition for a Banach lattice algebra to be representable as $C(K)$ with K compact.

S. J. Bernau

MR1106817 (92g:47048) 47B65 47B07 47D03
Martínez, Josep (E-VLNC-AN);
Mazón, José M. [Mazón Ruiz, José M.] (E-VLNC-AN)
Quasi-compactness of dominated positive operators and C_0-semigroups.
Math. Z. **207** (1991), *no. 1,* 109–120.

One of the most interesting problems in the theory of positive operators is the following. If $S, T\colon E \to E$ are two positive operators on a Banach lattice such that $0 \leq S \leq T$, then what are the properties of T that are inherited by S? In connection with compactness properties this problem has been studied extensively in the past [see P. G. Dodds and D. H. Fremlin, Israel J. Math. **34** (1979), no. 4, 287–320; MR0570888 (81g:47037); O. Burkinshaw and the reviewer, Math Z. **174** (1980), no. 3, 289–298; MR0593826 (81m:47053); Trans. Amer. Math. Soc. **283** (1984), no. 1, 369–381; MR0735429 (85e:47025)]. In this paper the authors study this domination problem when the operator T is quasi-compact, i.e., when there exist a compact operator K and some n such that $\|T^n - K\| < 1$. Here are three sample results: (1) If T is quasi-compact and $r(T) < 1$, then S is also quasi-compact. (2) If E is also Dedekind complete and $\sigma(T) = \sigma_0(T)$ (where $\sigma_0(T)$ is the spectrum of T in the algebra $\mathcal{L}^{\rm r}(E)$ of all regular operators), then $r_{\rm ess}(S) \leq r_{\rm ess}(T)$. (3) Let $E = L_p(\mu)$ with $1 < p \leq \infty$ and let $\{T(t)\}_{t \geq 0}$ and $\{S(t)\}_{t \geq 0}$ be positive C_0-semigroups of operators with generators A and B, respectively, such that $0 \leq S(t) \leq T(t)$ for each $t \geq 0$. If each $T(t)$ is quasi-compact and the semigroup $\{S(t)\}_{t \geq 0}$ is uniformly bounded, then each $S(t)$ is quasi-compact.

For proofs and more results we refer the reader to the interesting paper.

C. D. Aliprantis

MR780352 (87g:47076) 47B55 46A40 47D20
McPolin, P. T. N.; **Wickstead, A. W.** (4-QUEEN)
Relatively central operators on Archimedean vector lattices. III.
Quart. J. Math. Oxford Ser. (2) **36** (1985), *no. 141,* 75–89.

For the discussion below H will be a cofinal vector sublattice of an Archimedean vector lattice E. The vector space of relatively central operators is defined by $Z(E|H) = \{T: E \to H\colon \text{there exists } \lambda > 0 \text{ such that } x \in E,\ h \in H \text{ and } |x| \le |h| \Rightarrow |Tx| \le \lambda|h|\}$. The vector space $Z(E|H)$ is a partially ordered algebra and it has been studied by the authors in part II [see the preceding review]. In this paper they continue their investigation of relatively central operators in connection with some other types of operators.

The authors first present some examples to establish that $Z(E|H)$ is not a vector lattice, in general. Afterwards they consider operators $T: E \to H$ and introduce the notions of relatively ideal preserving operators (for all $x \in E$ there exists $\lambda_x > 0$ such that $h \in H$ and $|x| \le |h|$ imply $|Tx| \le \lambda_x|h|$), relatively band preserving operators ($h \in H$, $x \in E$ and $x \perp h$ imply $Tx \perp h$), and relative orthomorphisms. The authors present conditions under which these new operators are actually relatively central. For instance, they show that if E is a normed vector lattice, then every bounded relatively ideal preserving operator is relatively central.

In another direction, the authors commence an investigation of the relationship between projections and relatively central operators. Also, they study the extremal structure of certain sets of relatively central operators. *C. D. Aliprantis*

MR832971 (87g:47075) 47B55 46A40 47D20
McPolin, P. T. N.; **Wickstead, A. W.** (4-QUEEN)
Relatively central operators on Archimedean vector lattices. II.
J. Austral. Math. Soc. Ser. A **40** (1986), *no. 3,* 287–298.

Let H be a cofinal vector sublattice of an Archimedean vector lattice E. In part I [Proc. Roy. Irish Acad. Sect. A **80** (1980), no. 2, 191–208; MR0620620 (82g:47027)] Wickstead introduced and studied the collection $Z(E|H)$ of all (linear) operators $T\colon E\to H$ which have the property that there exists some $\lambda>0$ such that $x\in E$, $h\in H$ and $|x|\le|h|$ imply $|Tx|\le\lambda|h|$. The vector space $Z(E|H)$ is a partially ordered vector space under its natural ordering and it is an algebra under composition.

In the present paper the authors investigate the structure of $Z(E|H)$ as an algebra under composition. In particular, they study the relationship between the properties of having an identity, being abelian and being semisimple. If H is Dedekind complete in its own right, then they show that the preceding properties are equivalent. The authors also study spectral properties of operators in $Z(E|H)$. If E is a Banach lattice and H is closed, then for a nonzero operator $T\in Z(E|H)$, its spectrum relative to $Z(E|H)$ (which in this case is a Banach space) is the same as that of $T|_H$ relative to the center $Z(H)$ of H, and that of T relative to $\mathcal{L}(E)$. The limitations of the theorems are illustrated by examples and counterexamples, and some open problems are posed.

{For part III see the following review.} *C. D. Aliprantis*

MR1041484 (91e:46060) 46H05 46H20
Miller, John Boris (5-MNSH)
The natural ordering on a strictly real Banach algebra.
Math. Proc. Cambridge Philos. Soc. **107** (1990), *no. 3,* 539–556.

The author considers real Banach algebras in which every element has real spectrum. Such algebras are commutative modulo the radical, by a theorem of I. Kaplansky [Duke Math. J. **16** (1949), 399–418; MR0031193 (11,115d)]. The main result shows that the principal component of the group of invertible elements is a generating convex cone consisting of the elements with strictly positive spectra. This cone is used to define a partially ordered algebra structure and the corresponding open-interval topology. This topology is compared with the norm and spectral radius seminorm topologies. The existence of square roots of positive elements is established together with continuity of this operation with respect to the open-interval topology. The dual cone of positive functions is also studied. *Jaroslav Zemánek*

MR1961634 (2004d:47074) 47B60 46H05 47A10 47B65
Mouton, S. [Mouton, Sonja] (SA-STEL)
Convergence properties of positive elements in Banach algebras. (English summary)
Math. Proc. R. Ir. Acad. **102A** (2002), *no. 2,* 149–162.

The author studies qualitative properties of the peripheral spectrum of a positive operator (Riesz points, poles of the resolvent) and the possibility of their extension to positive approximants of the operator. The corresponding continuity properties of the Laurent series coefficients are also considered. *Jaroslav Zemánek*

References

1. F. Aràndiga and V. Caselles, Approximations of positive operators and continuity of the spectral radius, *Journal of Operator Theory* **26** (1991), 53–71. MR1214920 (94d:47035)
2. F. Aràndiga and V. Caselles, Approximations of positive operators and continuity of the spectral radius II, *Mathematische Zeitschrift* **209** (1992), 547–58. MR1156436 (94d:47036)
3. F. Aràndiga and V. Caselles, Approximations of positive operators and continuity of the spectral radius III, *Journal of the Australian Mathematical Society, Series A* **57** (1994), 330–40. MR1297007 (95j:47045)
4. W. Arendt, On the *o*-spectrum of regular operators and the spectrum of measures, *Mathematische Zeitschrift* **178** (1981), 271–87. MR0631633 (83h:47027)
5. B. Aupetit, Inessential elements in Banach algebras, *Bulletin of the London Mathematical Society* **18** (1986), 493–7. MR0847990 (87k:46100)
6. B. Aupetit, *A primer on spectral theory,* Springer-Verlag, New York–Heidelberg–Berlin, 1991. MR1083349 (92c:46001)
7. B. Aupetit and H. du T. Mouton, Spectrum preserving linear mappings in Banach algebras, *Studia Mathematica* **109** (1) (1994), 91–100. MR1267714 (95c:46070)
8. F.F. Bonsall and J. Duncan, *Complete normed algebras,* Springer-Verlag, New York–Heidelberg–Berlin, 1973. MR0423029 (54 #11013)
9. J.B. Conway, *A course in functional analysis,* Springer-Verlag, New York–Heidelberg–Berlin, 1990. MR1070713 (91e:46001)
10. J.J. Grobler and H. Raubenheimer, Spectral properties of elements in different Banach algebras, *Glasgow Mathematical Journal* **33** (1991), 11–20. MR1089949 (92a:46054)
11. J. Martinez and J.M. Mazón, Quasi-compactness of dominated positive operators and C_0-semigroups, *Mathematische Zeitschrift* **207** (1991), 109–20. MR1106817 (92g:47048)
12. H. du T. Mouton, On inessential ideals in Banach algebras, *Quaestiones Mathematicae* **17** (1994), 59–66. MR1276008 (95f:46080)
13. S. Mouton (née Rode) and H. Raubenheimer, More spectral theory in ordered Banach algebras, *Positivity* **1** (1997), 305–17. MR1660397 (2000a:46070)
14. G.K. Pedersen, *C^*-algebras and their automorphism groups,* Academic Press, London–New York, 1978. MR0548006 (81e:46037)
15. J. Puhl, The trace of finite and nuclear elements in Banach algebras, *Czechoslovak Mathematical Journal* **28** (1978), 656–76. MR0506439 (81a:47024)
16. H. Raubenheimer and S. Rode, Cones in Banach algebras, *Indagationes Mathemat-

icae n.s. **7** (4) (1996), 489–502. MR1620116 (99i:46035)
17. W. Rudin, *Functional analysis,* McGraw-Hill, New York, 1973. MR0365062 (51 #1315)
18. H.H. Schaefer, *Banach lattices and positive operators,* Springer-Verlag, New York–Heidelberg, 1974. MR0423039 (54 #11023)
19. H.H. Schaefer, On the *o*-spectrum of order bounded operators, *Mathematische Zeitschrift* **154** (1977), 79–84. MR0470748 (57 #10494)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1962967 (2004d:47075) 47B60 46H05 46H30 47A60 47B65
Mouton, S. [Mouton, Sonja]
A spectral problem in ordered Banach algebras. (English summary)
Bull. Austral. Math. Soc. **67** (2003), *no. 1,* 131–144.

The author studies the problem of positivity of the holomorphic functional calculus on positive elements. This is motivated by a particular case, the long-standing problem, of when a positive operator with spectrum consisting of 1 only will necessarily majorize the identity. An analogue of the Perron-Frobenius eigenvalue is found for the lower bound of the spectrum of a positive operator with respect to a certain inverse-closed cone. *Jaroslav Zemánek*

References

1. B. Aupetit, *A primer on spectral theory* (Springer-Verlag, New York, Heidelberg, Berlin, 1991). MR1083349 (92c:46001)
2. F.F. Bonsall and J. Duncan, *Complete normed algebras* (Springer-Verlag, New York, Heidelberg, Berlin, 1973). MR0423029 (54 #11013)
3. J.J. Grobler and C.B. Huijsmans, 'Doubly Abel bounded operators with single spectrum', *Quaestiones Math.* **18** (1995), 397–406. MR1354120 (96i:47001)
4. M. Mbekhta and J. Zemánek, 'Sur le théorème ergodique uniforme et le spectre', *C.R. Acad. Sci. Paris Sér. I Math.* **317** (1993), 1155–1158. MR1257230 (95b:47010)
5. H. du T. Moutou and S. Mouton, 'Domination properties in ordered Banach algebras', *Studia Math.* **149** (2002), 63–73. MR1881716 (2003e:46076)
6. S. Mouton, 'Convergence properties of positive elements in Banach algebras', *Proc. Roy. Irish Acad. Sect. A* (to appear). cf. MR 2004d:47074
7. S. Mouton (née Rode) and H. Raubenheimer, 'More spectral theory in ordered Banach algebras', *Positivity* **1** (1997), 305–317. MR1660397 (2000a:46070)
8. H. Raubenheimer and S. Rode, 'Cones in Banach algebras', *Indag. Math. (N.S.)* **7** (1996), 489–502. MR1620116 (99i:46035)
9. H.H. Schaefer, 'Some spectral properties of positive linear operators', *Pacific J. Math.* **10** (1960), 1009–1019. MR0115090 (22 #5893)
10. H.H. Schafer, M. Wolf and W. Arendt, 'On lattice isomorphisms with positive real spectrum and groups of positive operators', *Math. Z.* **164** (1978), 115–123. MR0517148 (80b:47048)
11. X.-D. Zhang, 'Some aspects of the spectral theory of positive operators', *Acta Appl. Math.* **27** (1992), 135–142. MR1184885 (93j:47056)
12. X.-D. Zhang, 'On spectral properties of positive operators', *Indag. Math. (N.S.)* **4** (1993), 111–127. MR1213328 (94b:47047)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2239814 (2007c:46047) 46H05 06F25 47B65
Mouton, S. [Mouton, Sonja]
On spectral continuity of positive elements. (English summary)
Studia Math. **174** (2006), *no. 1,* 75–84.

The author proves the continuity of the spectral radius of positive elements in ordered Banach algebras. An ordered Banach algebra is a Banach algebra A with a cone, that is, a set $C \subset A$ which is closed under addition, multiplication and scalar multiplication by nonnegative scalars. Clearly, the relation $x \le y$ if $y - x \in C$ defines an order on A. C is called normal if there exists $\alpha > 0$ such that $\|x\| \le \alpha\|y\|$, whenever $0 \le x \le y$ in A. For any element $a \in A$, $\varrho(x)$ denote the spectral radius of x, $\delta(x)$ denote the distance of spectrum of x from 0 in $\mathbb{C}$ and $A(x) = \{y \in A: x \le y, xy \le yx$ or $yx \le xy$ and $d(\varrho(x), \mathrm{Sp}(x)) \ge d(\alpha, \mathrm{Sp}(x))$ for all $\alpha \in \mathrm{Sp}(y)\}$. The author proves that if C is closed and normal, then for $x \in C$ and $y \in A(x)$, $\mathrm{Sp}(y) \subset \mathrm{Sp}(x) + \varrho(x - y)$. The author generalizes the inequality $\varrho(x + y) \le \varrho(x) + \varrho(y)$ for commuting elements x, y of any Banach algebra A to the setup when A is an ordered Banach algebra with normal cone C and for $x, y \in C$ satisfy $xy \le yx$ or $yx \le xy$. Besides a couple of results for spectral radius in ordered Banach algebra A, the author also proves the following. Let C be normal in an ordered Banach algebra, $x \in C$ and $y \in A$ be such that $x \le y$, $xy \le yx$ or $yx \le xy$, and $(\alpha 1 - x)^{-1} \in C$ for all $\alpha \in \mathrm{Sp}(y) \setminus \mathrm{Sp}(x)$. Then $\mathrm{Sp}(y) \subset \mathrm{Sp}(x) + \varrho(x - y)$.

Dinesh Jayantilal Karia

References

1. B. Aupetit, *A Primer on Spectral Theory*, Springer, New York, 1991. MR1083349 (92c:46001)
2. L. Burlando, *Continuity of spectrum and spectral radius in Banach algebras*, in: Functional Analysis and Operator Theory, J. Zemánek (ed.), Banach Center Publ. 30, Inst. Math., Polish Acad. Sci., Warszawa, 1994, 53–100. MR1285600 (95i:46062)
3. A. R. Esajan [A. R. Esayan], *On estimating the spectrum of the sum of positive semi-commuting operators*, Sibirsk. Mat. Zh. 7 (1966), 460–464 (in Russian); English transl.: Siberian Math. J. 7 (1966), 374–378. MR0194893 (33 #3099)
4. H. du T. Mouton and S. Mouton, *Domination properties in ordered Banach algebras*, Studia Math. 149 (2002), 63–73. MR1881716 (2003e:46076)
5. S. Mouton, *A spectral problem in ordered Banach algebras*, Bull. Austral. Math. Soc. 67 (2003), 131–144. MR1962967 (2004d:47075)
6. S. Mouton, *Convergence properties of positive elements in Banach algebras*, Math. Proc. R. Ir. Acad. Sect. A 102 (2002), 149–162. MR1961634 (2004d:47074)
7. S. Mouton (née Rode) and H. Raubenheimer, *More spectral theory in ordered Banach algebras*, Positivity 1 (1997), 305–317. MR1660397 (2000a:46070)
8. H. Raubenheimer and S. Rode, *Cones in Banach algebras*, Indag. Math. (N.S.) 7 (1996), 489–502. MR1620116 (99i:46035)
9. H. H. Schaefer, *Some spectral properties of positive linear operators*, Pacific J. Math. 10 (1960), 1009–1019. MR0115090 (22 #5893)
10. V. Ja. Stecenko [V. Ya. Stetsenko], *On a spectral property of an indecomposable operator*, Uspekhi Mat. Nauk 22 (1967), no. 3, 242–244 (in Russian). MR0215116 (35 #5959)
11. M. Zima, *On the local spectral radius in partially ordered Banach spaces*, Czechoslovak

Math. J. 49 (124) (1999), 835–841. MR1746709 (2001m:47011)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2260492 (2007g:46072) 46H05 06F25 46H10 47A10
Mouton, S. [Mouton, Sonja] (SA-STEL)
On the boundary spectrum in Banach algebras. (English summary)
Bull. Austral. Math. Soc. **74** (2006), *no. 2,* 239–246.

Let A be a complex Banach algebra with unit, and let S be the set of invertible elements of A. This article considers various properties of ∂S, the topological boundary of S, as well as properties of the boundary spectrum, defined by $S_\partial(a) = \{\lambda \in \mathbf{C}: \lambda - a \in \partial S\}$. Particular attention is paid to the case where A is an ordered Banach algebra.

{Reviewer's remark: The last equality in Theorem 2.15 appears to be superfluous, since, if T is an isomorphism, then its kernel must be zero.} *Thomas Ransford*

References

1. B. Aupetit, *A primer on spectral theory* (Springer-Verlag, New York, 1991). MR1083349 (92c:46001)
2. F.F. Bonsall and J. Duncan, *Complete normed algebras* (Springer-Verlag, New York, 1973). MR0423029 (54 #11013)
3. P.R. Halmos, *A Hilbert space problem book* (Springer-Verlag, New York, 1982). MR0675952 (84e:47001)
4. H. du T. Mouton and S. Mouton, 'Domination properties in ordered Banach algebras', *Studia Math.* **149** (2002), 63–73. MR1881716 (2003e:46076)
5. S. Mouton, 'Convergence properties of positive elements in Banach algebras', *Math. Proc. R. Ir. Acad.* **102A** (2002), 149–162. MR1961634 (2004d:47074)
6. S. Mouton, 'A spectral problem in ordered Banach algebras', *Bull. Austral. Math. Soc.* **67** (2003), 131–144. MR1962967 (2004d:47075)
7. S. Mouton, 'On spectral continuity of positive elements', *Studia Math.* **174** (2006), 75–84. MR2239814 (2007c:46047)
8. S. Mouton (née Rode) and H. Raubenheimer, 'More spectral theory in ordered Banach algebras', *Positivity* **1** (1997), 305–317. MR1660397 (2000a:46070)
9. H. Raubenheimer and S. Rode, 'Cones in Banach algebras', *Indag. Math. (N.S.)* **7** (1996), 489–502. MR1620116 (99i:46035)
10. A.E. Taylor and D.C. Lay, *Introduction to functional analysis* (Krieger, Florida, 1986). MR0862116 (87k:46001)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2470837 (2009m:46072) 46H05
Mouton, S. [Mouton, Sonja] (SA-STEL)
A condition for spectral continuity of positive elements. (English summary)
Proc. Amer. Math. Soc. **137** (2009), *no. 5,* 1777–1782.

Summary: "Let a be an element of a Banach algebra A. We introduce a compact subset $T(a)$ of the complex plane, show that the function which maps a onto $T(a)$ is upper semicontinuous and use this fact to provide a condition on a which ensures that if (a_n) is a sequence of positive elements converging to a, then the sequence of the spectral radii of the terms a_n converges to the spectral radius of a in the case in which A is partially ordered by a closed and normal algebra cone and a is a positive element."

References

1. B. Aupetit, *A Primer on Spectral Theory*, Springer, New York, 1991. MR1083349 (92c:46001)
2. L. Burlando, *Continuity of spectrum and spectral radius in Banach algebras*, in: Functional Analysis and Operator Theory, J. Zem'anek (ed.), Banach Center Publ. 30, Inst. Math., Polish Acad. Sci., Warsaw, 1994, 53–100. MR1285600 (95i:46062)
3. L. Burlando, *Noncontinuity of spectrum for the adjoint of an operator*, Proc. Amer. Math. Soc. **128** (2000), 173–182. MR1625705 (2000c:47007)
4. J. B. Conway and B. B. Morrel, *Operators that are points of spectral continuity*, Integral Equations Operator Theory **2** (1979), 174–198. MR0543882 (80h:47004)
5. S. V. Djordjevi'c and Y. M. Han, *Browder's theorems and spectral continuity*, Glasg. Math. J. **42** (2000), 479–486. MR1793814 (2001h:47003)
6. S. V. Djordjevi'c and Y. M. Han, *Spectral continuity for operator matrices*, Glasg. Math. J. **43** (2001), 487–490. MR1878591 (2002k:47008)
7. P. R. Halmos, *A Hilbert Space Problem Book*, Graduate Texts in Math., vol. 19, Springer, New York, 1982. MR0675952 (84e:47001)
8. H. du T. Mouton and S. Mouton, *Domination properties in ordered Banach algebras*, Studia Math. **149** (2002), 63–73. MR1881716 (2003e:46076)
9. S. Mouton, *A spectral problem in ordered Banach algebras*, Bull. Austral. Math. Soc. **67** (2003), 131–144. MR1962967 (2004d:47075)
10. S. Mouton, *Convergence properties of positive elements in Banach algebras*, Math. Proc. R. Ir. Acad. Sect. A **102** (2002), 149–162. MR1961634 (2004d:47074)
11. S. Mouton, *On spectral continuity of positive elements*, Studia Math. **174** (2006), 75–84. MR2239814 (2007c:46047)
12. S. Mouton, *On the boundary spectrum in Banach algebras*, Bull. Austral. Math. Soc. **74** (2006), 239–246. MR2260492 (2007g:46072)
13. S. Mouton (n'ee Rode) and H. Raubenheimer, *More spectral theory in ordered Banach algebras*, Positivity **1** (1997), 305–317. MR1660397 (2000a:46070)
14. G. J. Murphy, *Continuity of the spectrum and spectral radius*, Proc. Amer. Math. Soc. **82** (1981), 619–621. MR0614889 (82h:46066)
15. J.D. Newburgh, *The variation of spectra*, Duke Math. J. **18** (1951), 165–176. MR0051441 (14,481b)
16. H. Raubenheimer and S. Rode, *Cones in Banach algebras*, Indag. Math. (N.S.) **7** (1996), 489–502. MR1620116 (99i:46035)
17. H. H. Schaefer, *Some spectral properties of positive linear operators*, Pacific J. Math.

10 (1960), 1009–1019. MR0115090 (22 #5893)

Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR1881716 (2003e:46076) 46H05 46B40 47A10 47B65
Mouton, H. du T. (SA-STEL-EEL); **Mouton, S.** [**Mouton, Sonja**] (SA-STEL)
Domination properties in ordered Banach algebras. (English summary)
Studia Math. **149** (2002), *no. 1,* 63–73.

An ordered Banach algebra A has a cone A^+ which is closed under multiplication, and the cone is said to be "normal" if there is $k>0$ for which $0 \leq a \leq b \Longrightarrow \|a\| \leq \|b\|$. This guarantees that the cone is "proper", in the sense that $A^+ \cap -A^+ = \{0\}$, and also that the spectral radius $|\cdot|_\sigma$ is "monotone", in the sense that $0 \leq a \leq b \Longrightarrow |a|_\sigma \leq |b|_\sigma$. In this note the authors address "domination properties": If $0 \leq a \leq b$ then which properties of b are transmitted to a? Membership of the radical of A is one such property; the authors show that this is true under each of a number of auxiliary conditions on the cone A^+.

Robin Harte

References

1. C. D. Aliprantis and O. Burkinshaw, *Dunford-Pettis operators on Banach lattices*, Trans. Amer. Math. Soc. 274 (1982), 227–238. MR0670929 (84b:47045)
2. C. D. Aliprantis, O. Burkinshaw, *On weakly compact operators on Banach lattices*, Proc. Amer. Math. Soc. 83 (1981), 573–587. MR0627695 (82j:47057)
3. C. D. Aliprantis, O. Burkinshaw, *Positive compact operators on Banach lattices*, Math. Z. 174 (1980), 289–298. MR0593826 (81m:47053)
4. B. Aupetit, *A Primer on Spectral Theory*, Springer, New York, 1991. MR1083349 (92c:46001)
5. V. Caselles, *On the peripheral spectrum of positive operators*, Israel J. Math. 58 (1987), 144–160. MR0901175 (88j:47054)
6. P. G. Dodds and D. H. Fremlin, *Compact operators in Banach lattices*, ibid. 34 (1979), 287–320. MR0570888 (81g:47037)
7. W. K. Hayman and P. B. Kennedy, *Subharmonic Functions I*, London Math. Soc. Monogr. 9, Academic Press, London, 1976. MR0460672 (57 #665)
8. S. Mouton (née Rode) and H. Raubenheimer, *More spectral theory in ordered Banach algebras*, Positivity 1 (1997), 305–317. MR1660397 (2000a:46070)
9. H. Raubenheimer and S. Rode, *Cones in Banach algebras*, Indag. Math. (N.S.) 7 (4) (1996), 489–502. MR1620116 (99i:46035)
10. A. C. Zaanen, *Riesz Spaces II*, North-Holland, Amsterdam, 1983. MR0704021 (86b:46001)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1660397 (2000a:46070) 46H05 46B40 47A10 47B60
Mouton, S. [Mouton, Sonja] (SA-STEL);
Raubenheimer, H. [Raubenheimer, Heinrich] (SA-OFS)
More spectral theory in ordered Banach algebras. (English summary)
Positivity **1** (1997), *no. 4,* 305–317.

In a recent paper Raubenheimer and S. Rode [Indag. Math. (N.S.) **7** (1996), no. 4, 489–502; MR1620116 (99i:46035)] introduced their concept of an "ordered Banach algebra", having a positive cone A^+ closed under unrestricted multiplication; they showed that if this cone is also topologically closed and "normal" then the Perron-Frobenius theorem holds; the spectral radius of a positive element is included in the spectrum: $0 \leq a \in A \implies |a|_\sigma \in \sigma(a)$. Here they turn their attention to the Kreĭn-Rutman theorem, and show (Theorem 3.2) that if the spectral radius is a pole of the resolvent then it is also an eigenvalue, simultaneous for both left and right multiplication, with a positive eigenvector. This is the case if for example $a \in A$ is "Riesz" with respect to an "inessential" ideal $J \subseteq A$; in this situation they find (Theorem 4.1) that, provided the quotient cone $(A/J)^+$ also gives "spectral radius monotonicity", the "peripheral spectrum" of $a \in A$ consists of isolated points. We might remark that all this seems to work if the positive cone is only required to contain the product of commuting pairs of its elements. *Robin Harte*

MR0220061 (36 #3128) 46.50 06.00
Ng, Kung-fu
A representation theorem for partially ordered Banach algebras.
Proc. Cambridge Philos. Soc. **64** 1968 53–59

Let A be a real algebra that is also a Banach space. Continuity and commutativity of multiplication are not assumed. Let U denote the unit ball. Suppose that A is ordered by a strict semialgebra A^+, and that U is order-convex. Then there is an isometric, order-preserving isomorphism of A with a space $C_0(X)$ (X locally compact, Hausdorff) if and only if the following conditions are satisfied: (i) U is directed by the ordering; (ii) if $a, b \in A^+ \cap U$, then $ab \leq a, b$; (iii) for each $a \in A^+$, there exist sequences $\{b_n\}, \{c_n\}$ in $A^+ \cap U$ such that $a = \lim ab_n = \lim c_n a$. Simple examples are given to show that these conditions are independent. The result generalises M. H. Stone's theorem on ordered algebras with a multiplicative identity that is also an order-unit [Proc. Nat. Acad. Sci. U.S.A. **26** (1940), 280–283; MR0002023 (1,338e)]. *G. J. O. Jameson*

MR1456587 (98k:47073) 47B65 47A10 47A35
Räbiger, Frank (D-TBNG-MI); **Wolff, Manfred P. H.** (D-TBNG-MI)
Spectral and asymptotic properties of dominated operators. (English summary)
J. Austral. Math. Soc. Ser. A **63** (1997), *no. 1,* 16–31.

Let E be a Banach lattice and let T be a positive linear operator from E into E. Let S be a linear operator dominated by T, i.e., $|S(x)| \leq T|x|$ for all $x \in E$. The authors study the relationship between the peripheral spectra of T and S. In case S is also positive they obtain, e.g., the result that if T satisfies the growth condition (G), then $\sigma(S) \cap r(T)\Gamma \subseteq \sigma(T) \cap r(T)\Gamma$, where Γ denotes the unit circle in the complex plane. The analogous result for the point spectra is obtained under the hypothesis that T is Abel ergodic or that E is a KB-space. Assuming that $r(T)$ is a Riesz point of T, the authors prove that for any dominated operator S the essential spectral radius $r_{\text{ess}}(S)$ satisfies the strict inequality $r_{\text{ess}}(S) < r(T)$. In the final section the authors study the inheritance of some asymptotic properties such as almost periodicity, strong convergence of the powers.

{See also the following review [MR1446832 (98k:47074)].} *Anton Schep*

MR1738035 (2001b:47009) 47A10 47B60 47D06
Räbiger, Frank (D-TBNG-MI); **Wolff, Manfred P. H.** (D-TBNG-MI)
Spectral and asymptotic properties of resolvent-dominated operators. (English summary)
J. Austral. Math. Soc. Ser. A **68** (2000), *no. 2,* 181–201.

This is a continuation of the authors' earlier paper [J. Austral. Math. Soc. Ser. A **63** (1997), no. 1, 16–31; MR1456587 (98k:47073)], which was concerned with bounded operators. Here they study pseudo-resolvents, and more particularly unbounded operators A and B on a complex Banach lattice E such that B is resolvent-dominated by A in the sense that $|(s-B)^{-1}x| \leq (s-A)^{-1}|x|$ for all $x \in E$ and all sufficiently large real s. This requires that A is resolvent-positive in the sense of W. Arendt [Proc. London Math. Soc. (3) **54** (1987), no. 2, 321–349; MR0872810 (88c:47074)], and in particular $s(A) < \infty$, where $s(A) = \sup\{\mathrm{Re}\,\lambda\colon\ \lambda \in \sigma(A)\}$.

A typical result is that if B is resolvent-positive and resolvent-dominated by A and $(s-A)^{-1}$ satisfies a certain growth condition as $s \downarrow s(A)$, then $\sigma(B) \cap (s(A) + i\mathbf{R}) \subseteq \sigma(A)$. There are similar results concerning the peripheral part of the point spectrum, the essential spectrum and quasi-compactness. The results are applied to the generators of C_0-semigroups, showing that various spectral conditions on a positive semigroup imply stability, ergodicity or almost periodicity of dominated semigroups. *C. J. K. Batty*

References

1. C. D. Aliprantis and O. Burkinshaw, *Positive operators* (Academic Press, Orlando, 1985). MR0809372 (87h:47086)
2. F. Andreu and J. M. Mazon, 'On the boundary spectrum of dominated C_0-semigroups', *Semigroup Forum* **38** (1989), 129–139. MR0976198 (90a:47095)
3. W. Arendt, 'Kato's inequality. A characterization of generators of positive semigroups', *Proc. Roy. Irish Acad.* **84** (1984), 155–174. MR0790308 (86j:47057)
4. W. Arendt, 'Resolvent positive operators', *Proc. London Math. Soc. (3)* **54** (1987), 321–349. MR0872810 (88c:47074)
5. W. Arendt and C. J. K. Batty, 'Domination and ergodicity for positive semigroups', *Proc. Amer. Math. Soc.* **114** (1992), 743–747. MR1072082 (92f:47038)
6. W. Arendt and C. J. K. Batty, 'Absorption semigroups and Dirichlet boundary conditions', *Math. Ann.* **292** (1993), 427–448. MR1204830 (94c:47065)
7. W. Arendt and A. Rhandi, 'Perturbation of positive semigroups', *Arch. Math.* **56** (1991), 107–119. MR1086478 (91k:47089)
8. C. J. K. Batty and Vũ Quôc Phóng, 'Stability of individual elements under one-parameter semigroups', *Trans. Amer. Math. Soc.* **322** (1990), 805–818. MR1022866 (91c:47072)
9. Ph. Clément, H. J. A. M. Heijmans, S. Angenent, C. J. van Duijn and B. de Pagter, *One-parameter semigroups* (North-Holland, Amsterdam, 1987). MR0915552 (89b:47058)
10. E. Yu. Emel'yanov, U. Kohler, F. Räbiger and M. P. H. Wolff, 'Stability and almost periodicity of dominated semigroups of operators', preprint.
11. J. Esterle, E. Strouse and F. Zouakia, 'Stabilité asymptotique de certains semigroupes d'opérateurs et ideaux primaires', *J. Operator Theory* **28** (1992), 203–228. MR1273043 (95f:43001)
12. G. Greiner, 'Zur Perron-Frobenius Theorie stark stetiger Halbgruppen', *Math. Z.*

177 (1981), 401–423. MR0618205 (82i:47063)

13. G. Greiner, 'Spekrum und Asymptotik stark stetiger Halbgruppen positiver Operatoren', *Sitzungsber. Heidelb. Akad. Wiss., Math-Naturwiss. Klasse* (1982), 55–80. MR0669010 (84j:47069)
14. E. Hille and R. S. Phillips, *Functional analysis and semi-groups,* Colloquium Publications 31 (Amer. Math. Soc., Providence, 1957). MR0089373 (19,664d)
15. U. Krengel, *Ergodic theorems* (deGruyter, Berlin, 1985). MR0797411 (87i:28001)
16. L. H. Loomis, *An introduction to abstract harmonic analysis* (Van Nostrand, Princeton, 1953). MR0054173 (14,883c)
17. J. Martinez and J. M. Mazon, 'Quasi-compactness of dominated positive operators and C_0-semigroups', *Math. Z.* **207** (1991), 109–120. MR1106817 (92g:47048)
18. P. Meyer-Nieberg, *Banach lattices* (Springer, Berlin, 1991). MR1128093 (93f:46025)
19. R. Nagel (ed.), *One-parameter semigroups of positive operators* (Springer, Berlin, 1986). MR0839450 (88i:47022)
20. F. Räbiger, 'Stability and ergodicity of dominated semigroups, I. The uniform case', *Math. Z.* **214** (1993), 43–54. MR1234596 (94i:47066)
21. F. Räbiger and M. P. H. Wolff, 'On the approximation of positive operators and the behaviour of the spectra of the approximants', *Integral Equations Operator Theory* **28** (1997), 72–86. MR1446832 (98k:47074)
22. F. Räbiger and M. P. H. Wolff, 'Spectral and asymptotic properties of dominated operators', *J. Austral. Math. Soc. (Ser. A)* **63** (1997), 16–31. MR1456587 (98k:47073)
23. H. H. Schaefer, *Banach lattices and positive operators* (Springer, Berlin, 1974). MR0423039 (54 #11023)
24. A. R. Schep, 'Weak Kato-inequalities and positive semigroups', *Math. Z.* **190** (1985), 303–314. MR0806890 (86k:47040)
25. S.-Y. Shaw, 'Uniform ergodic theorems for locally integrable semigroups and pseudo-resolvents', *Proc. Amer. Math. Soc.* **98** (1986), 61–67. MR0848876 (87j:47015)
26. J. Voigt, 'Absorption semigroups, their generators, and Schrödinger semigroups', *J. Funct. Anal.* **67** (1982), 167–205. MR0845197 (88a:81036)
27. Vũ Quôc Phóng, 'Theorems of Katznelson-Tzafriri type for semigroups of operators', *J. Funct. Anal.* **103** (1992), 74–84. MR1144683 (93e:47050)
28. Vũ Quôc Phóng and Yu. I. Lyubich, 'A spectral criterion for almost periodicity of one-parameter semigroups', *J. Soviet. Math.* **48** (1990), 644–647. MR0916441 (89a:47067)
29. K. Yosida, *Functional analysis,* 6th edition (Springer, Berlin, 1980). MR0617913 (82i:46002)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR771655 (86c:47050) 47B55 47A10
Raubenheimer, Heinrich (SA-OFS)
The o-spectrum of r-asymptotically quasifinite-rank operators.
Quaestiones Math. **7** (1984), *no. 3,* 299–303.

The author introduces the class of r-asymptotically quasifinite-rank operators (which properly contains the class of r-compact operators introduced by W. Arendt [Math. Z. **178** (1981), no. 2, 271–287; MR0631633 (83h:47027)]). If E is a complex Banach lattice, and if T is a regular operator on E, then T is said to be r-asymptotically quasifinite-rank if $[\Omega_r(T^n)]^{1/n} \to 0$ as $n \to \infty$, where $\Omega_r(T) = \inf\{\|T - C\|_r\colon C \in E' \otimes E\}$. (Here $\| \, \|_r$ denotes the usual r-norm on the space $L^r(E)$ of all regular operators on E.) The principal result of the paper is that the spectrum and the o-spectrum coincide for such operators.

Peter Dodds

MR857772 (87m:47090) 47B55 47B05
Raubenheimer, Heinrich (SA-OFS)
r-asymptotically quasifinite rank operators and the spectrum of measures.
Quaestiones Math. **10** (1986), *no. 1,* 97–111.

The operators of the title were previously introduced by the author [same journal **7** (1984), no. 3, 299–303; MR0771655 (86c:47050)] and, in the present paper, are characterized as those regular linear mappings T on a complex Banach lattice E for which $\lambda - T$ is invertible in the space of regular operators on E modulo the ideal of finite rank operators, for each nonzero complex λ. The author gives several applications to the spectral analysis of convolution operators on compact groups. These are closely related to earlier work of W. Arendt [Math. Z. **178** (1981), no. 2, 271–287; MR0631633 (83h:47027)].
Peter Dodds

MR1795734 (2002e:47021) 47B06 47B65
Raubenheimer, Heinrich (SA-OFS)
On regular Riesz operators. (English summary)
Quaest. Math. **23** (2000), *no. 2,* 179–186.

Let E be a Banach lattice and let $\mathcal{L}_r(E)$ denote the regular operators on E. By $\mathcal{K}_r(E)$ we denote the r-compact operators on E, i.e., $T \in \mathcal{K}_r(E)$ if T is a limit of finite-rank operators in the regular norm. An operator is now said to be r-asymptotically of finite rank if $T + \mathcal{K}_r(E)$ is quasi-nilpotent in $\mathcal{L}_r(E)/\mathcal{K}_r(E)$. One of the first results of this paper is that an operator T is r-asymptotically of finite rank if and only if it is a Riesz operator and the order spectrum of T equals the spectrum of T. Another result is that T is r-asymptotically of finite rank if and only if T' is r-asymptotically of finite rank. Then the author indicates some partial results concerning the domination problem of positive Riesz operators. If $0 \leq S \leq T$ and T is a Riesz operator such that the order spectrum of T equals the spectrum of T, then S is a Riesz operator. It remains an open question whether this holds without the assumption on the equality of the two spectra.

Anton Schep

References

1. C.D. Aliprantis and O. Burkinshaw, Positive compact operators in Banach lattices, *Math. Z.* **174** (1980), 289–298. MR0593826 (81m:47053)
2. W. Arendt, On the o-spectrum and the spectrum of measures, *Math. Z.* **178** (1981), 271–287. MR0631633 (83h:47027)
3. W. Arendt and A.R. Sourour, Perturbation of regular operators and the order essential spectrum, *Indag. Mathem.* **89** (1986), 109–122. MR0849713 (87j:47052)
4. B. Aupetit, *A primer on spectral theory,* Springer-Verlag, 1991. MR1083349 (92c:46001)
5. B.A. Barnes, G.J. Murphy, M.R.F. Smyth and T.T. West, *Research notes in mathematics 67. Riesz and Fredholm theory in Banach algebras,* Pitman, 1982. MR0668516 (84a:46108)
6. V. Caselles, On the peripheral spectrum of positive operators, *Israel J. Math.* **58** (1987), 144–160. MR0901175 (88j:47054)
7. J.B. Conway, *Functions of one complex variable,* Springer-Verlag 1978. MR0503901 (80c:30003)
8. J.B. Conway, *A course in functional analysis,* Springer-Verlag, 1990. MR1070713 (91e:46001)
9. B. de Pagter and A.R. Schep, Measures of non-compactness of operators in Banach lattices, *J. Funct. Anal.* **78** (1988), 31–55. MR0937631 (89d:47079)
10. P.G. Dodds and D.H. Fremlin, Compact operators in Banach lattices, *Israel J. Math.* **34** (1979), 287–320. MR0570888 (81g:47037)
11. H.R. Dowson, *Spectral theory of linear operators,* Academic Press, 1978. MR0511427 (80c:47022)
12. J.J. Grobler and H. Raubenheimer, Spectral properties of elements in different Banach algebras, *Glasgow Math. J.* **33** (1991), 11–20. MR1089949 (92a:46054)
13. H.P. Lotz, Über das Spektrum positiver Operatoren, *Math. Z.* **108** (1968), 15–32. MR0240648 (39 #1994)
14. J. Martinez and J.M. Mazón, Quasi-compactness of dominated positive opera-

tors and C_0-semigroups, *Math. Z.* **207** (1991), 109–120. MR1106817 (92g:47048)
15. P. Meyer-Nieberg, *Banach lattices,* Springer-Verlag, 1991. MR1128093 (93f:46025)
16. S. Mouton and H. Raubenheimer, More spectral theory in ordered Banach algebras, *Positivity* **1** (1997), 305–317. MR1660397 (2000a:46070)
17. H. Raubenheimer, The o-spectrum of r-asymptotically quasi finite rank operators, *Quaestiones Math.* **7** (1984), 299–303. MR0771655 (86c:47050)
18. H. Raubenheimer, r-Asymptotically quasi finite rank operators and the spectrum of measures, *Quaestiones Math.* **10** (1986), 97–111. MR0857772 (87m:47090)
19. H. Raubenheimer and S. Rode, Cones in Banach algebras, *Indag. Mathem.* **7** (1996), 489–502. MR1620116 (99i:46035)
20. H.H. Schaefer, *Banach lattices and positive operators,* Springer-Verlag, 1974. MR0423039 (54 #11023)
21. H.H. Schaefer, On the o-spectrum of order bounded operators, *Math. Z.* **154** (1977), 79–84. MR0470748 (57 #10494)
22. E. Scheffold, Das Spektrum von Verbandsoperatoren in Banachverbänden, *Math. Z.* **123** (1971), 177–190. MR0324475 (48 #2827)
23. A.C. Zaanen, *Introduction to operator theory in Riesz spaces,* Springer-Verlag, 1991. MR1631533 (2000c:47074)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1620116 (99i:46035) 46H20 46B40
Raubenheimer, H. [Raubenheimer, Heinrich] (SA-OFS);
Rode, S. [Mouton, S.] (SA-OFS)
Cones in Banach algebras. (English summary)
Indag. Math. (N.S.) **7** (1996), *no. 4,* 489–502.

The Perron-Frobenius theorem says that the spectral radius of a positive matrix is in the spectrum, and this extends to positive operators on Banach lattices [H. H. Schaefer, *Banach lattices and positive operators*, Springer, New York, 1974; MR0423039 (54 #11023)]. In the theory of positive operators the "positive cone" is usually induced from a positive cone on the underlying space, and properties such as "normality" are expected to follow from corresponding properties of the cone on the space. Here the authors work in the algebra of operators, making the simplest assumptions about the positive cone $A^+ \subseteq A$, in particular that $A^+A^+ \subseteq A^+$, i.e. the unrestricted product of positive elements, is positive, and inferring (Theorem 4.1) that the spectral radius is "monotone": that is, $0 \le a \le b$ implies $|a|_\sigma \le |b|_\sigma$. This spectral radius monotonicity in turn yields (Theorem 5.2) the Perron-Frobenius result that $0 \le a$ implies $|a|_\sigma \in \sigma(a)$: the argument is by contradiction involving Stirling's formula, and taken from B. de Pagter and A. R. Schep [J. Funct. Anal. **78** (1988), no. 1, 31–55; MR0937631 (89d:47079)(Prop. 3.3)]. This is indeed very neat, even if the unrestricted positivity is a bit strong, excluding for example the usual positivity for Hilbert space operators and C^*-algebras. Thus if we only assume the implication $0 \le a, 0 \le b, ba = ab \Longrightarrow 0 \le ab$ then we get "commuting spectral radius monotonicity", which is enough for the Perron-Frobenius theorem. In fact the authors here do obtain the C^* case (Theorem 6.5): to achieve commuting spectral radius monotonicity when $ab - ba = 0 \le a \le b$ consider the closed C^* subalgebra generated by $a = a^*$ and $b = b^*$, whose commutativity does not even need Fuglede's theorem. *Robin Harte*

MR1156625 (93c:46095) 46J10 46B42 46E05
Render, H. (D-DUIS)
Lattice structures of ordered Banach algebras.
Illinois J. Math. **36** (1992), *no. 2,* 238–250.

Conditions are considered for a Banach algebra A with approximate identity and ordered by a multiplicative cone to be isomorphic to a sublattice and subalgebra of $C(X)$, the space of continuous real-valued functions on a compact space. It is shown that a partially ordered (real) vector space is a vector lattice if and only if it satisfies a disjoint decomposition property (DDP), i.e., each z can be written as $x-y$ for x and y positive and disjoint ($[0,x]\cap[0,y]=\{0\}$), and the Riesz decomposition property. For A with closed multiplicative cone, it is established that the following conditions (and some others) are equivalent: A possesses the DDP and products of disjoint positive elements are zero; A is an almost f-algebra (a lattice and products of disjoint positive elements are zero); A is isomorphic to a sublattice (and subalgebra) of $C(X)$. If A has a closed multiplicative cone containing all squares then A is isomorphic to a subalgebra of $C(X)$. Consequences of this and uniqueness of the positive cone are considered.

W. A. Feldman

MR2736139 (2012c:15069) 15B48 15A69 15A75 34C12 47A15 47L07
Sanchez, Luis A. [Sánchez, Luis Ángel] (E-UPCT-ACE)
Convex cones associated to generalized cones in $\mathbb{R}^N$. (English summary)
Linear Algebra Appl. **433** (2010), *no. 11-12,* 2122–2138.

Applications of algebra (e.g. groups, rings, ideals, fields, modules, vector spaces, tensor spaces and other extensions) are the cornerstone of modern mathematics. A priori, the axiomatic definitions of a given set of algebraic structures often exploit this notion. For example, multilinear algebra, viz. the theory of k-linear alternate forms, has invoked the small vibration analysis of mechanical systems [F. R. Gantmakher and M. G. Kreĭn, *Oszillationsmatrizen, Oszillationskerne und kleine Schwingungen mechanischer Systeme*, Wissenschaftliche Bearbeitung der deutschen Ausgabe: Alfred Stöhr. Mathematische Lehrbücher und Monographien, I. Abteilung, Bd. V, Akademie Verlag, Berlin, 1960; MR0114338 (22 #5161)] and the theory of totally positive matrices, non-negative matrices and associated linear algebraic systems [T. Ando, Linear Algebra Appl. **90** (1987), 165–219; MR0884118 (88b:15023); A. Berman and R. J. Plemmons, *Nonnegative matrices in the mathematical sciences*, revised reprint of the 1979 original, Classics Appl. Math., 9, SIAM, Philadelphia, PA, 1994; MR1298430 (95e:15013)]. In fact, the consideration of linear algebra from a modern perspective sheds light on the analysis of certain differential equations, involving the basic concept of monotonicity and positivity theory of monotone systems [M. W. Hirsch and H. L. Smith, in *Handbook of differential equations: ordinary differential equations. Vol. II*, 239–357, Elsevier B. V., Amsterdam, 2005; MR2182759 (2006j:37017); H. L. Smith, *Monotone dynamical systems*, Math. Surveys Monogr., 41, Amer. Math. Soc., Providence, RI, 1995; MR1319817 (96c:34002)]. Interestingly, dynamical behavior, in the light of positive operators, explicates a set of direct consequences of Perron-Frobenius theory. For a given set of positive eigenvectors, the consideration of the existence of dominant eigenvalues reveals the spectral properties of arbitrary cones of rank k. In this sense, the notion of a positive operator led the author to provide a generalized Perron-Frobenius theory [L. Á. Sánchez, J. Differential Equations **246** (2009), no. 5, 1978–1990; MR2494695 (2010b:37067)]. From the perspective of totally positive operators, this is a new geometric insight into the classical operator theory. The author has introduced [op. cit.] the notion of generalized cones for a new class of monotone systems, as a special case of the present consideration, namely, the fact that the cones of rank 2 possess a Poincaré-Bendixson property.

In this setting, the present paper extends the notion of periodic orbits of the above-mentioned two-dimensional classical setting to the case of arbitrary finite dimensions. Thereby, a number of interesting applications of vector space theory are intertwined with matrix theory, operator theory, dynamical systems and function analysis. Namely, this paper examines multilinear properties of certain convex cones associated to the generalized Euclidean cones: it offers the underlying spectral properties of generalized cones of rank k in $\mathbb{R}^N$ associated to convex cones in the space of alternate k-linear forms over $\mathbb{R}^N$. The consideration of the present work offers far-reaching consequences in the theory of generalized positive operators, such as the proof of a Perron-Frobenius theorem. Namely, it is shown here that the notions of convex cones and generalized cones have a strong connection with the stability of periodic orbits. For a given cone of rank k in $\mathbb{R}^N$, the author has constructed a convex cone K in the space of k-linear alternating forms over $\mathbb{R}^N$ such that the underlying positive operators induce standard positivity with respect to the cone K. This provides a new geometric proof of the

Perron-Frobenius results [G. Fusco and W. M. Oliva, Ann. Mat. Pura Appl. (4) **160** (1991), 63–76 (1992); MR1163201 (95c:15042); M. A. Krasnosel′skiĭ, E. A. Lifshits and A. V. Sobolev, *Positive linear systems*, translated from the Russian by Jürgen Appell, Sigma Ser. Appl. Math., 5, Heldermann, Berlin, 1989; MR1038527 (91f:47051)].

In detail, this paper describes the beautiful connection of total positivity to the skew-symmetric product of a given set of vectors, and gives a broad generalization of the theory of totally positive matrices through its combination with the geometrical setup of positive linear operators. From the perspective of k-linear alternating forms, the author provides a set of purely algebraic properties. This offers a wider explanation of the geometrical and topological properties that underlie the theory of convex cones. The third section of the present paper is devoted to the cone of rank $k \in \mathbb{N}$. Namely, it constructs the associated convex cones, in the space of alternate k-linear forms Λ_k, from the basic definitions and noteworthy properties of convex cones. In Section 4, the author provides the theory of generalized positive operators and shows that it induces positive operators with respect to associated convex cones in Λ^k. This offers a new proof of the classical Perron-Frobenius theorem of positive operators. In Section 5, the author gives a very well-known example of a generalized cone defined as the number of sign changes of the concerned coordinate vectors in $\mathbb{R}^N$. Although such a consideration already exists in the literature [T. Ando, op. cit.; F. R. Gantmakher and M. G. Kreĭn, op. cit.; E. A. Lifshits and A. V. Sobolev, op. cit.], at least at some level of abstraction, from the viewpoint of previous work the author has reformulated those results in order to fit the existing considerations into a generic approach. Finally, the author closes his consideration by showing a set of prospective applications in stability theory, periodic orbits and new autonomous systems [L. Á. Sánchez, J. Math. Anal. Appl. **317** (2006), no. 1, 71–79; MR2205312 (2006j:34087); Nonlinear Anal. Real World Appl. **10** (2009), no. 4, 2151–2156; MR2508426 (2010d:34109); B. Schwarz, Pacific J. Math. **32** (1970), 203–229; MR0257466 (41 #2117)]. *Bhupendra Nath Tiwari*

References

1. T. Ando, Totally positive matrices, Linear Algebra Appl. 90 (1987) 165–219. MR0884118 (88b:15023)
2. A. Berman, R.J. Plemmons, Nonnegative Matrices in Mathematical Sciences, SIAM, Philadelphia, 1994. MR1298430 (95e:15013)
3. V.A. Boichenko, G.A. Leonov, V. Reitmann, Dimension Theory for Ordinary Differential Equations, Teubnes Texts in Mathematics, vol. 141, Stuttgart, 2005. MR2381409 (2009i:37057)
4. G. Fusco, M.W. Oliva, A Perron theorem for the existence of invariant subspaces, Ann. Mat. Pura Appl. 160 (1991) 63–76. MR1163201 (95c:15042)
5. F.R. Gantmacher, M.G. Krein, Oszillationsmatrizen, Oszillationskerne und kleine Schwingungen mechanischer Systeme, Akademie-Verlar, Berlin, 1960. MR0114338 (22 #5161)
6. M.W. Hirsch, H.L. Smith, Monotone Dynamical Systems, Handbook of Differential Equations: Ordinary Differential Equations, vol. II, Elsevier B.V., Amsterdam, 2005, pp. 239–357. MR2182759 (2006j:37017)
7. M.A. Krasnoselskij, J.A. Lifshits, A.V. Sobolev, Positive Linear Systems, Heldermann Verlag, Berlin, 1989. MR1038527 (91f:47051)
8. L.A. Sanchez, Dynamics of the modified Michaelis-Menten system, J. Math. Anal. Appl. 317 (2006) 71–79. MR2205312 (2006j:34087)
9. L.A. Sanchez, Global asymptotic stability of the Goodwin system with repression, Nonlinear Anal. Real World Appl. 10 (2009) 2151–2156. MR2508426 (2010d:34109)

10. L.A. Sanchez, Cones of rank 2 and the Poincaré-Bendixson property for a new class of monotone systems, J. Differential Equations 216 (2009) 1170–1190. MR2494695 (2010b:37067)
11. B. Schwarz, Totally positive differential systems, Pacific J. Math. 32 (1970) 203–229. MR0257466 (41 #2117)
12. H.L. Smith, Monotone Dynamical Systems, American Mathematical Society, Providence, 1995. MR1319817 (96c:34002)
13. R.H. Wasserman, Tensors & Manifolds, Oxford University Press, Oxford, 1992. MR1178055 (93h:53002)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1213325 (94e:47045) 47B38 43A22 47A10
Saxe, Karen (1-MACA)
Essential order spectra of convolution operators. (English summary)
Indag. Math. (N.S.) **4** (1993), *no. 1,* 79–89.

Given an arbitrary complex Borel measure $\mu \in M(G)$ on a locally compact group G and any $1 \leq p \leq \infty$, let $T_{\mu,p}$ denote the corresponding convolution operator on $L^p(G)$. It is well known that its spectrum $\sigma(T_{\mu,p})$ is always contained in the spectrum $\sigma(\mu)$ of μ in the measure algebra $M(G)$, but examples show that this inclusion may be proper. On the other hand, for amenable G, W. Arendt has shown that $\sigma(\mu)$ coincides with the order spectrum of the operator $T_{\mu,p}$, i.e. the spectrum of $T_{\mu,p}$ in the Banach algebra of all regular operators on the Banach lattice $L^p(G)$ [see Math. Z. **178** (1981), no. 2, 271–287; MR0631633 (83h:47027)].

In the paper under review, similar results are obtained for certain essential spectra of such convolution operators. The main result is Theorem 2.4 which states that, for any complex Borel measure μ on a compact group G, the essential spectrum, the Weyl spectrum, and the Browder spectrum of μ in the Banach algebra $M(G)$ are all identical and also coincide with the order essential spectrum, the order Weyl spectrum, and the order Browder spectrum of the convolution operator $T_{\mu,p}$ for any $1 \leq p \leq \infty$. The latter terms have a canonical meaning which is carefully explained in the paper. Moreover, a number of interesting facts concerning these essential spectra are presented in a general setting, mainly based on the monograph by B. A. Barnes et al. [*Riesz and Fredholm theory in Banach algebras*, Pitman, Boston, MA, 1982; MR0668516 (84a:46108)].

Michael M. Neumann

MR0115090 (22 #5893) 46.00
Schaefer, Helmut
Some spectral properties of positive linear operators.
Pacific J. Math. **10** 1960 1009–1019

Let E be a partially ordered Banach space with positive cone K, T a positive operator (bounded linear operator in E that maps K into itself), r the spectral radius of T, and R_λ its resolvent operator. The author proves that certain topological conditions on K imply properties of the spectrum, spectral radius, and resolvent of an arbitrary positive operator. Also, by imposing conditions on the positive operator stronger conclusions are obtained. A positive operator T is said to be quasi-interior if there exists $\lambda > r$ such that $TR_\lambda x$ is a quasi-interior point of K for every non-zero point x of K. Among other results it is proved that if T is quasi-interior and r is a pole of R_λ, then (1) $r > 0$ and r is a simple pole of R_λ, and (2) every characteristic vector of T in K corresponding to r is quasi-interior to K. Certain additional conditions are given which imply that the nullspace of $rI - T$ has dimension one. *F. F. Bonsall*

MR0218912 (36 #1996) 47.25 28.00
Schaefer, H. H.
Invariant ideals of positive operators in $C(X)$. I.
Illinois J. Math. **11** 1967 703–715

Let X be a compact Hausdorff space, $C(X)$ the algebra of continuous complex-valued functions on X, and T a positive linear operator on $C(X)$. In this paper the author begins a systematic study of the closed T-invariant ideals of $C(X)$, called T-ideals, using (among others) spectral techniques drawn from the work of M. G. Kreĭn and M. A. Rutman [Uspehi Mat. Nauk **3** (1948), no. 1 (23), 3–95; MR0027128 (10,256c); translated in Amer. Math. Soc. Transl. No. 26 (1950); MR0038008 (12,341b)] and the author's own earlier work on ordered spaces and spectral operators [e.g., Math. Z. **82** (1963), 303–313; MR0205081 (34 #4916)].

Theorem 1: Every maximal T-ideal is of the form $I_\varphi = (f \in C(X) : \varphi(|f|) = 0\}$, for a suitable normalized eigenvector φ of the adjoint operator T'. Here 0 is the eigenvalue for φ if and only if φ is a point measure concentrated at a point of X where Te vanishes (e is the identity of $C(X)$).

T is called ergodic if for each $f \in C(X)$ the closed convex hull of the orbit of f (under the powers of T) contains a function invariant under T. T is called a Markov operator if $Te = e$. Theorem 2: If T is ergodic and Markov, and if Φ is the set of all positive normalized T-invariant measures on X, then $\varphi \to I_\varphi$ is a bijection of the set of all extreme points of Φ onto the family of all maximal T-ideals. Every T-ideal of the form I_φ is the intersection of the maximal T-ideals containing it. "Ergodic" cannot be dropped from the hypotheses in this theorem.

The author gives examples, subsidiary comments and corollaries, including a dual to Theorem 2 concerning stochastic operators on a space $L^1(\mu)$. *R. A. Raimi*

MR0233216 (38 #1539) 47.25
Schaefer, H. H.
Invariant ideals of positive operators in $C(X)$. II.
Illinois J. Math. **12** 1968 525–538

Let X be a compact Hausdorff space, $C(X)$ the Banach algebra of continuous complex-valued functions on X, and T a positive linear operator on $C(X)$. This paper is the second part of a study of the ideals in $C(X)$ that are invariant under T, and continues the nomenclature, numeration and bibliography of Part I [same J. **11** (1967), 703–715; MR0218912 (36 #1996)]; this review will do the same.

The intersection of all maximal T-ideals is called the T-radical, R; if $R=(0)$, T is called radical-free. Denote $n^{-1}(I+T+\cdots+T^{n-1})$ by Mn. The main theorem of Section 4 says that if T is ergodic and Markovian, then $f\in R$ if and only if $\lim_n Mn|f|=0$. As a corollary (under the same hypotheses), a sufficient condition for T to be radical-free is that there exist a strictly positive T-invariant measure on X. This condition is necessary if X is also metrizable.

In Section 5 the author collects and adds to known results concerning the peripheral spectrum of T, that part of the spectrum lying on the circle of radius $r(T)$, the spectral radius (with a similar definition for the peripheral point spectrum). T is said to be uniformly ergodic if Mn is a Cauchy sequence in norm, and irreducible if (0) is the only T-ideal. Some theorems are as follows: If T is uniformly ergodic and $\|T\|=r(T)=1$, then each element of the peripheral spectrum of T is an eigenvalue of the second adjoint of T. If T is irreducible and Markovian, the peripheral spectrum is a subgroup of the circle group.

In Section 6, T is assumed to be weakly compact but no longer Markovian or ergodic. A number $\rho\geq 0$ is called a distinguished eigenvalue of the adjoint T' if T' has a positive eigenvector for ρ. Theorem: Suppose that $T\geq 0$ is weakly compact and radical-free. Then, for each distinguished eigenvalue of T', the corresponding eigenspace is a vector sublattice of the dual space of $C(X)$, and these sublattices are mutually orthogonal. Moreover, each positive normalized eigenvector of T' is a unique convex combination (barycenter) of those (normalized, positive) eigenvectors that belong to the same eigenvalue and determine maximal T-ideals. *R. A. Raimi*

MR0470748 (57 #10494) 47B55 47A10

Schaefer, Helmut H.

On the o-spectrum of order bounded operators.

Math. Z. **154** (1977), *no. 1,* 79–84.

The author discusses the difference between spectra of a regular operator calculated in the algebra of regular operators and in the algebra of bounded operators, respectively. The setting is that of a complex Banach vector lattice. *S. Kutateladze*

MR565135 (81f:47038) 47B55 47D05
Schaefer, H. H.
Ordnungsstrukturen in der Operatorentheorie. (German)
Jahresber. Deutsch. Math.-Verein. **82** (1980), *no. 1,* 33–50.

This is a survey lecture given by the author at a meeting of the German Mathematical Society in Hamburg. Its contents give a rather complete idea about the material considered. (1) Operator ideals. (1.1) Preliminaries, (1.2) Order-bounded operators, (1.3) Cone absolutely summing and majorized operators, (1.4) Integral operators. (2) The theory of the spectrum. (2.1) Preliminaries, (2.2) The symmetry of the peripheral spectrum, (2.3) Irreducible and peripheral point-spectrum, (2.4) Groups of positive operators. (3) One-parameter semigroups. (3.1) Preliminaries, (3.2) The resolvent of a generator A, (3.3) The spectrum of A, (3.4) Characterization of A. The proofs are omitted.

{Reviewer's remark: It is worthwhile to mention that the paper of A. K. Kitover [Dokl. Akad. Nauk SSSR **250** (1980), no. 4, 800–803; MR0560379 (81d:47028)] makes a contribution to the topic of Section 2 and that V. Ja. Stecenko [Uspehi Mat. Nauk **22** (1967), no. 3(135), 242–244; MR0215116 (35 #5959)] proved a theorem which is very close to Theorem 2D (due to Niiro and Sawashima).} *Yu. A. Abramovich*

MR666257 (84j:47071) 47D05 47B55 47D10
Schaefer, H. H.

Some recent results on positive groups and semigroups.

From A to Z (Leiden, 1982), pp. 69–79, *Math. Centre Tracts*, 149, *Math. Centrum*, *Amsterdam*, 1982.

The classical Perron-Frobenius theory describes properties of the spectrum of positive matrices (i.e. matrices with nonnegative entries). This theory has been extended to positive operators on a Banach lattice [H. H. Schaefer, *Banach lattices and positive operators*, see Chapter V, Springer, New York, 1974; MR0423039 (54 #11023)]. More recently, the spectral theory of a single positive operator has been applied to groups of positive operators and generalized to one-parameter semigroups of positive operators. The article under consideration gives a survey of this development. In Section 1 the situation for a single operator is reviewed briefly. Spectral properties of groups of positive operators are discussed in Section 2. As an application, estimates for the norm-distance of distinct elements of the group are given. In Section 3, finally, some results are selected out of the Perron-Frobenius theory of one-parameter semigroups of positive operators. The presentation parallels that of the single operator (Section 1); this underlines the unity of the basic ideas involved, although there are considerable differences in the methods of proof. The article is of an expository nature, making the theory accessible to the nonspecialist. Results and basic ideas are illuminated by the consideration of special cases like positive operators on $C(K)$ or matrices. The specialist will find new aspects and applications as well as a list of open problems.

{For the entire collection see MR0666249 (83h:46007)}

W. Arendt (Zbl 486:47024)

MR517148 (80b:47048) 47B55 47D10
Schaefer, Helmut H.; **Wolff, Manfred [Wolff, Manfred P. H.]**; **Arendt, Wolfgang**
On lattice isomorphisms with positive real spectrum and groups of positive operators.
Math. Z. **164** (1978), *no. 2,* 115–123.

The main result is the following theorem. Let T be a lattice isomorphism of a Banach lattice E. The following two statements are equivalent: (i) T is in the center $Z(E)$, i.e., $|Tx| \leq n(T)|x|$ for all $x \in E$ and some $n(T) \in \mathbf{N}$; (ii) the spectrum $\sigma(T)$ of T consists of positive real numbers only. An immediate corollary shows that if T is a lattice homomorphism satisfying $\sigma(T) = \{1\}$, then T is the identity on E. Some interesting applications are given. *A. V. Bukhvalov*

MR0324475 (48 #2827) 47B55
Scheffold, Egon
Das Spektrum von Verbandsoperatoren in Banachverbänden. (German)
Math. Z. **123** (1971), 177–190.

Seien E ein reeller Banachverband und $T \in L(E)$ ein Verbandsoperator auf E (d.h. es gilt $|Tx| = T|x|$ für alle $x \in E$). In natürlicher Weise werde T auf die Komplexifizierung $\tilde{E}$ von E fortgesetzt, auf der durch $|z| := \sup\{(\cos\alpha)x + (\sin\alpha)y \colon 0 \le \alpha \le 2\pi\}$ und $\|z\| := \||z|\|$ für $z = x + iy \in \tilde{E}$ ein Absolutbetrag und eine Norm erklärt werden. Der Autor untersucht das Spektrum solcher Operatoren und zeigt u.a., daß die Verbandsoperatoren ein zyklisches Spektrum besitzen. Umgekehrt existiert zu jeder kompakten zyklischen Menge der komplexen Ebene C ein Verbandsoperator, der diese Menge als Spektrum besitzt ($A \subset C$ heißt zyklisch, wenn aus $\alpha = |\alpha|\gamma \in A$ folgt $|\alpha|\gamma^k \in A$ für alle ganzen k). Falls E ein Banachverband stetiger Funktionen ist, kann diese Charakterisierung verschärft werden. Außerdem werden Bedingungen für die Zyklizität des peripheren Spektrums von T und T' angegeben, wenn T aus der umfassenderen Klasse der positiven Operatoren aus $L(E)$ stammt bzw. wenn T ein positives Element einer kommutativen Banachverbandsalgebra mit Einselement ist. *H. Schwetlick*

MR581429 (81m:46064) 46H20 46B30
Scheffold, Egon
Über komplexe Banachverbandsalgebren. **(German. English summary)**
J. Funct. Anal. **37** (1980), *no. 3,* 382–400.

A real Banach lattice algebra A is simultaneously a real Banach lattice and a real algebra satisfying $0 \leq xy$ and $\|xy\| \leq \|x\| \cdot \|y\|$ for $0 \leq x,\ y \in A$. The author studies complex Banach lattice algebras which are complexifications of real Banach lattice algebras and he aims for a Gel′fand theory for such spaces. It turns out that this is possible as soon as the absolute values of the continuous multiplicative linear forms are again multiplicative. In that case the spectrum of A is shown to be cyclic. *R. J. Nagel*

MR612873 (82f:46060) 46J35 46B30
Scheffold, Egon
FF-Banachverbandsalgebren. (German)
Math. Z. **177** (1981), *no. 2,* 193–205.

A real Banach lattice algebra (B.1.a.) A is a real Banach lattice and an associative algebra over $\mathbf{R}$ such that $xy \geq 0$ and $\|xy\| \leq \|x\|\|y\|$ for positive elements $x, y \in A$. A complex B.1.a. is the complexification of such a real algebra with $\|z\| = \||z|\|$, where $|z| = \sup\{(\cos\alpha)x + (\sin\alpha)y \colon 0 \leq \alpha < 2\pi\}$, $z = x + iy$. An FF-algebra satisfies moreover the condition that $\inf(a, b) = 0$ implies $ab = 0$ for $a, b \in A$. In Section 1 the author studies $C_{\mathbf{R}}(X)$, X a compact Hausdorff space, as an FF-algebra with some nonpointwise multiplications. In Section 2 it is shown, among other results, that FF-algebras are commutative, that they are algebraically and lattice isomorphic to function B.1.a.'s in the case when they are semisimple, and that radicals in such algebras are given by $\operatorname{rad} A = \{x \in A \colon x^3 = 0\}$. In Section 3 it is shown that unital B.1.a.'s are algebraically, norm, and lattice isomorphic to B.1.a.'s of type $C(X)$. *W. Żelazko*

MR733772 (85h:46070) 46H10 46B30
Scheffold, Egon (D-DARM)
Banachverbandsalgebren mit einer natürlichen Wedderburn-Zerlegung.
(German) **[Banach lattice algebras with a natural Wedderburn decomposition]**
Math. Z. **185** (1984), *no. 4,* 521–531.

A real Banach lattice algebra A is a real Banach lattice and an associative algebra such that $x,\ y\ge 0$ implies $xy\ge 0$ and $\|xy\|\le\|x\|\,\|y\|$. To each positive u in A there corresponds a principal lattice ideal $A_u=\bigcup_{n=1}^\infty\{x\in A\colon |x|\le nu\}$ which is lattice isomorphic to $C_R(K_u)$, where K_u is a compact Hausdorff space. It is assumed that A as a Banach algebra has the identity e. In the first section the author discusses the role of this assumption and proves Theorem 1, that A_e is a subalgebra of A isometrically isomorphic to $C_R(K_e)$ also as a real Banach algebra. One also has $A=A_e\oplus(A_e)^\perp$ (Theorem 3). If $C_R(K)$ is a unital Banach lattice algebra under some multiplication $*$, then $(f*g)(t)=(1/e(t))f(t)g(t)$ (Theorem 4). In Section 2 the author deals with the lattice $(A_e)^\perp$. He considers the following condition $\mathcal{O}$: $\inf(|a|,|b|)=0$ implies $r(ab)=0$, where r is the spectral radius and a and b are arbitrary elements in A. Put $N^*=\{x\in A\colon r(|x|)=0\}$. Then $(A_e)^\perp=N^*$ if and only if the condition $\mathcal{O}$ holds, and then $(A_e)^\perp$ is the Jacobson radical of A, so that Theorem 3 in this case gives a Wedderburn type decomposition.

W. Żelazko

MR788952 (86i:46056) 46J99 46B30
Scheffold, Egon (D-DARM)

Über den Spektralradius bei kommutativen Banachverbandsalgebren. (German) [On the spectral radius in commutative Banach lattice algebras]

Arch. Math. (Basel) **44** (1985), *no. 4,* 365–368.

Let A be a commutative real Banach lattice algebra with unit e ($e > 0$, $\|e\| = 1$). Previously the author had shown that A is an order direct sum of a lattice ideal A_e generated by e and its orthogonal completion $A_e^{\perp}$. The main result of the paper states that the following are equivalent: (i) $r(z) = r(|z|)$ for all z in $A_{\mathbf{C}}$, (ii) $r(z) = 0$ for all z in $(A_e^{\perp})_{\mathbf{C}}$. Here r is the spectral radius and $B_{\mathbf{C}}$ is the complexification of B. *W. Żelazko*

MR991863 (90i:46090) 46H10 46B30
Scheffold, E. (D-DARM)

Über Banachverbandsalgebren mit multiplikativer Zerlegungseigenschaft. (German) [On Banach lattice algebras with multiplicative decomposition property]

Acta Math. Hungar. **52** (1988), *no. 3-4,* 273–289.

Continuing his previous paper [J. Funct. Anal. **37** (1980), no. 3, 382–400; MR0581429 (81m:46064)] the author gives several further results on Banach lattice algebras with multiplicative decomposition property. They concern homomorphisms, maximal structure ideals and the decomposition property itself. *W. Żelazko*

MR1006654 (90f:46046) 46E25 46E05 46J25 46J35
Scheffold, Egon (D-DARM)

Über FF-Banachverbandsalgebren $C(K)$ und ihren assoziierten Operator. **(German. English summary)** **[On FF-Banach lattice algebras $C(K)$ and their associated operator]**

Rev. Roumaine Math. Pures Appl. **34** (1989), *no. 4,* 347–354.

This paper deals with a special type of multiplication on the Banach lattice of all real-valued continuous functions $C(K)$ on a compact Hausdorff space K. This nontrivial multiplication is denoted by $*$. Definition: Under this multiplication $C(K)$ is called an FF-Banach lattice algebra provided that the following four conditions hold: (i) $C(K)$ is a real associative algebra; (ii) $f*g\geq 0$ for all nonnegative $f,g\in C(K)$; (iii) $\inf(f,g)=0$ implies $f*g=0$ for $f,g\in C(K)$; and (iv) $\|f*g\|\leq\|f\|\,\|g\|$ for all $f,g\in C(K)$. The simplest example is $C(K)$ under the familiar pointwise multiplication of functions.

According to Theorem 1.2 of an earlier paper by the author [Math. Z. **177** (1981), no. 2, 193–205; MR0612873 (82f:46060)], the multiplication $*$ can be represented as follows: there exists a uniquely determined continuous map $t\mapsto\mu_t$ from K into the set $M(K)_+$ of nonnegative Radon measures on K, endowed with the topology induced by the topology $\sigma(M(K),C(K))$, such that, for all $f,g\in C(K)$, we have $f*g(t)=\int_k fg\,d\mu_t$ for all $t\in K$.

This gives rise to a positive linear operator T, $\|T\|\leq 1$, defined by $(Tf)(t)=\int_K f\,d\mu_t$ for all $f\in C(K)$ and $t\in K$. Note that $Tf=e_K*f$ for all $f\in C(K)$, where $e_K\equiv 1$ on K. Definition: The operator T described above is called the operator associated with the FF-Banach lattice algebra $(C(K),*)$. Note that $f*g=T(fg)$ for all $f,g\in C(K)$.

The author presents several results on this associated operator T. We need appropriate notation before describing a few such results. Set $U:=\{t\in K\colon \mu_t(e_K)>0\}$ and $B:=\bigcup_{t\in K}S_{\mu_t}$, where S_{μ_t} is the carrier of μ_t, and $U_0=B\cap U$.

Theorem 1: Let T be a positive endomorphism of $C(K)$ with $\|T\|\leq 1$. Further, let $\mu_t:=T'\varepsilon_t$ for all $t\in K$ (where ε_t denotes the Dirac point mass at t, and T' denotes the adjoint map). Through the equation $f*g:=T(f\cdot g)$ a multiplication $*$ is defined on $C(K)$, which makes $C(K)$ into an FF-Banach lattice algebra precisely if the operator T has the property (S) $\mu_t=\mu_t(e_K)\varepsilon_t$ for all $t\in B$.

Example: Define the positive endomorphism T on $C[0,2]$ as follows: $Tf(t)=(1-t)^{-1}\int_1^{2-t}f(u)\,du$ for $0\leq t<1$, $Tf(t)=(2-t)f(t)$, for $1\leq t\leq 2$. Then $B=[1,2]$ and T has property (S), so that $C[0,2]$ becomes an FF-Banach lattice algebra under the multiplication $f*g=T(fg)$.

A characterization of the regular maximal ideals in $(C(K),*)$—identified with the set M of all nontrivial real-valued linear multiplicative functionals—is given in the next theorem. Theorem 2: Let $U\cap B\neq\varnothing$; then $M=\{\mu_t(e_K)\varepsilon_t\colon t\in U\cap B\}$. Corollary 3: The radical $\mathcal{R}$ of $(C(K),*)$ consists of all functions that vanish identically on the set $U\cap B$.

The author also gives a characterization of the annihilator $\mathcal{A}$ of $(C(K),*)$, where $\mathcal{A}=\{f\in C(K)\colon f*C(K)=0\}$. Theorem 4: Let $f\in C(K)$; then $f\in\mathcal{A}$ if and only if $f\equiv 0$ on B.

Definition: A Banach lattice algebra A is called an F-Banach lattice algebra (note the single F) when $\inf(a,b)=0$ implies $\inf(c,a,b)=\inf(a,c,b)=0$ for all $a,b,c\in A$ and $c\geq 0$. Theorem 5: The quotient algebra $C(K)/\mathcal{A}$ is an F-Banach lattice algebra, and the quotient algebra $C(K)/\mathcal{R}$ is a semisimple F-Banach lattice algebra.

The next result describes the spectrum of T. Theorem 6: Let T be the operator associated with the FF-Banach lattice algebra $(C(K),*)$. Then: (i) The operator T is invertible precisely if $U=B=K$. In the case when T is invertible, $\sigma(T)=\{e_K * e_K(t)\colon\ t\in K\}$. (ii) If T is not invertible, then $\sigma(T)=\{0\}\cup\{e_K*e_K(t)\colon\ t\in\overline{B}\}$.

The final result of the paper is Theorem 7, whose exact statement we omit. This theorem shows that the double dual of $(C(K),*)$, furnished with the Arens product, is itself an FF-Banach lattice algebra, having T'' as its associated operator.

The interested reader should consider looking up the author's earlier paper [op. cit.]. With the exception of the result on the Arens product (Theorem 7), most results in the present paper are related to, and often dependent upon, results in the 1981 paper.

Christopher C. White

MR1124955 (92h:46027) 46B42 46H05 46J05
Scheffold, Egon (D-DARM)
Der Bidual von F-Banachverbandsalgebren. **(German)** **[The bidual of Banach F-algebras]**
Acta Sci. Math. (Szeged) **55** (1991), *no. 1-2,* 167–179.

The main result of the paper is the following: the norm (= order) bidual A'' of a Banach f-algebra A is again a Banach f-algebra with respect to the Arens multiplication. Recently, this result was generalized by the reviewer for the nonnormed case: If A is an f-algebra A with separating order dual A', then the order bidual A'', equipped with the Arens multiplication, is also an f-algebra [see the reviewer, J. Operator Theory **22** (1989), no. 2, 277–290; MR1043728 (91d:46061)]. The author also proves the following nice characterization: The only Banach f-algebras for which the second dual has an algebraic unit element of norm 1 are of type $C_0(X)$. *C. B. Huijsmans*

MR1213518 (94g:46052) 46J05 46B42
Scheffold, Egon (D-DARM)

Über den ordnungsstetigen Bidual von FF-Banachverbandsalgebren. **(German)**
[On the order-continuous bidual of FF-Banach lattice algebras]

Arch. Math. (Basel) **60** (1993), *no. 5,* 473–477.

A Banach lattice is called a Banach f-algebra if $\inf(a,b)=0$ implies $ab=0$. The main result states that the order continuous bidual $(A')'_n$ of a Banach f-algebra with the Arens product is a Banach f-algebra and therefore commutative. *A. Alexiewicz*

MR1239593 (95c:47042) 47B65 47B48
Scheffold, Egon (D-DARM)

Über Reynoldsoperatoren und "Mittelwert bildende" Operatoren auf halbeinfachen F-Banachverbandsalgebren. (German) [On Reynolds operators and averaging operators on semisimple F-Banach lattice algebras]

Math. Nachr. **162** (1993), 329–337.

Let A be a Banach lattice algebra which is also an f-algebra and assume that A is semisimple as a Banach algebra. Let T be a linear operator from A into A. Then T is called a Reynolds operator if $T(fTg+gTf)=TfTg+T(TfTg)$ for all $f,g\in A$ and T is called an averaging operator if $T(fTg)=TfTg$ for all $f,g\in A$. Let $r(g)$ denote the spectral radius of $g\in A$. Then the author proves in Theorem 2 of the paper that $r(Tg)\leq r(g)$ for all $g\in A$ for a Reynolds operator T, and in Theorem 3 of the paper that $r(Tg)\leq r(T)r(g)$ for all $g\in A$ for an averaging operator T. He then shows that the bi-adjoint of a Reynolds (or averaging) operator is again a Reynolds (or averaging) operator on the bidual A'' of A with respect to the Arens multiplication on A''. This result combined with Theorems 2 and 3 allows the author to extend a Reynolds (or averaging) operator T on A to a Reynolds (or averaging) operator on a $C(K)$-space, where K is a compact Hausdorff space. In the final section of the paper he then studies Reynolds operators on a $C(K)$-space and applies his extension to obtain a result on the existence of T-invariant ideals in A. *Anton Schep*

MR1315492 (96d:46071) 46J99 46B42 46H99
Scheffold, Egon (D-DARM)

Über Bimorphismen und das Arens-Produkt bei kommutativen D-Banachverbandsalgebren. (German. English summary) [On bimorphisms and the Arens product in commutative D-Banach lattice algebras]

Rev. Roumaine Math. Pures Appl. **39** (1994), *no. 3,* 259–270.

Summary: "Let E, F and G be Banach lattices and let $\varphi\colon E\times F\to G$ be bilinear and positive. In this paper we give a representation for the Arens triadjoint $\varphi^{***}\colon E''\times F''\to G''$ (in this context, $\varphi^*\colon G'\times E\to F'$; $\varphi^{**}\colon F''\times G''\to E'$). Without tensor products we show the following duality relation: φ is a lattice bimorphism if a certain map φ' is interval preserving. As an application the following result is shown: For a commutative d-Banach lattice algebra A (i.e. $|ab|=|a|\,|b|$ for $a,b\in A$) the order continuous part $(A')'_n$ of the bidual A'', equipped with the Arens product, is again a commutative d-Banach lattice algebra."

MR1459770 (2000b:46079) 46H05 43A10 46J99
Scheffold, E. (D-DARM)
Maßalgebren mit intervallerhaltender linksseitiger Multiplikation. (German)
[Measure algebras with interval-preserving left multiplication]
Acta Math. Hungar. **76** (1997), *no. 1-2,* 59–67.

A Banach lattice algebra A is a Banach lattice which, at the same time, is a Banach algebra such that the positive cone A^+ is invariant with respect to multiplication. Left multiplication on a Banach lattice algebra A is said to be [almost] interval preserving if $a \cdot [0, b]$ is equal to [resp., dense in] $[0, ab]$ for $a, b \in A^+$. An L-algebra is a Banach lattice algebra with an abstract L-space as the underlying Banach space. As an example, for L-algebras having interval-preserving left multiplication, measure algebras $M((S, \varphi))$ are introduced as follows: Let S be a compact Hausdorff space, $\varphi\colon S \to S$ continuous and satisfying $\varphi^2 = \varphi$, and define $s \cdot t := \varphi(t)$ $(s, t \in S)$. In this way (S, φ) becomes a compact topological semigroup. The corresponding convolution measure algebra $B := M((S, \varphi))$ (the dual of $C((S, \varphi))$ as a Banach space) is clearly an L-algebra and has the following properties (among which the first is immediate, and the remaining two are shown to hold in Theorem 3): (1) The norm is multiplicative on B^+; (2) left multiplication on B is interval preserving; (3) B is Arens regular, i.e. the two Arens products on its bidual coincide. The main result of this paper is that essentially (i.e. apart from taking suitable subobjects) there are no other L-algebras having properties (1) and (2), even if "almost interval preserving" is admitted. The crucial step towards this aim is the following representation theorem (Theorem 6): If A is an L-algebra having almost interval-preserving left multiplication and such that the norm is multiplicative on A^+, then its bidual, equipped with the first Arens product, is an $M((K, \varphi))$-measure algebra whose left multiplication is even interval preserving (A being an L-space, its dual is of the form $C(K)$ for some compact Hausdorff space K; φ is constructed in the proof of Theorem 6). This in turn leads to the desired characterization (Theorem 7): L-algebras having almost interval-preserving left multiplication and multiplicative norm on the positive cone are exactly the solid closed subalgebras of $M((K, \varphi))$-measure algebras; moreover, they are necessarily Arens regular and their left multiplication is interval preserving. Actually, only the hard part of this statement is to be found explicitly as Theorem 7; the (easy) converse is contained implicitly in the paper and has been added here by the reviewer. *Michael Grosser*

MR2011927 (2004h:46052) 46H99 46B42
Scheffold, Egon (D-DARM)

Über Banachverbandsalgebren vom Typ 1. **(German)** **[On Banach lattice algebras of type 1]**

Ark. Mat. **41** (2003), *no. 2,* 375–379.

A unital Banach lattice algebra A (with positive unit e of norm 1) is said to be of Type 1 if for every positive element a in A it follows that $a(e+a)^{-1}$ is positive. The result of this paper is the following. If in a Banach lattice algebra A of Type 1 every positive element is invertible then A is isomorphic to the real numbers. *Gerard Buskes*

References

1. Bonsall, F. F. und Duncan, J.,, *Complete Normed Algebras*, Springer-Verlag, Berlin—Heidelberg—New York, 1973. MR0423029 (54 #11013)
2. Schaefer, H. H., *Banach Lattices and Positive Operators*, Springer-Verlag, Berlin—Heidelberg—New York, 1974. MR0423039 (54 #11023)
3. Scheffold, E., Über komplexe Banachverbandsalgebren, *J. Funct. Anal.* **37** (1980), 382–400. MR0581429 (81m:46064)
4. Scheffold, E., Banachverbandsalgebren mit einer natürlichen Wedderburn-Zerlegung, *Math. Z.* **185** (1984), 521–531. MR0733772 (85h:46070)
5. Zhang, X.-D., On spectral properties of positive operators, *Indag. Math.* **4** (1993), 111–127. MR1213328 (94b:47047)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2097087 (2005i:47062) 47B65 46B42 46H99 47A10
Scheffold, E. (D-DARM)

Über positive Resolventenwerte positiver Operatoren. **(German. English summary)** **[On positive resolvent values of positive operators]**

Positivity **8** (2004), *no. 2,* 179–186.

This paper studies positive resolvent values of positive operators. Interestingly, some of its content plays out on the larger scale of Banach lattice algebras. Indeed, the present paper shows (Satz 4) that a positive invertible element x of a Banach lattice algebra A is a resolvent value of an element in A^+ if and only if it satisfies the so-called negative principle (das negative Prinzip), i.e. if $a \in A$ and $\lambda < 0$ and $xa \leq \lambda a$ then $xa \leq 0$. The latter has immediate consequences (see, e.g., Satz 5 of the present paper) for positive linear operators on Dedekind complete Banach lattices when one defines a positive linear operator T on a Banach lattice E to satisfy the negative principle if $T(x) \leq \lambda x$ for $\lambda \leq 0$ and $x \in E$ implies that $T(x) \leq 0$. That the negative principle plays an important role in spectral properties of resolvents of positive operators is also shown by Satz 1 and Satz 2 in the present paper.

In the last section of the paper the author looks at Banach lattices of the type $C_0(X)$. The author shows that a positive, invertible operator T on $C_0(X)$ is the resolvent value of a positive operator S on $C_0(X)$ if and only if T satisfies the following: $f \in C_0(X)$, $x_0 \in X$, $T(f) \geq 0$, and $T(f)(x_0) = 0$ implies $f(x_0) \leq 0$. The latter property is named the zero-minimum principle (das Prinzip vom Null-Minimum) by the author. For further details we refer the interested reader to the paper. *Gerard Buskes*

References

1. Johnson, C.R.: Inverse M-Matrices, *Linear Algebra and its Applications*, **47** (1982), 195–216. MR0672744 (83m:15003)
2. Nagel, R.J.: *One-parameter Semigroups of Positive Operators*, Lecture Notes Math. **1184**, Springer Verlag, Berlin, Heidelberg, New York, Tokyo, 1986. MR0839450 (88i:47022)
3. Schaefer, H.H.: *Topological vector spaces*, Springer Verlag, Berlin, Heidelberg, New York, 1971. MR0342978 (49 #7722)
4. Schaefer, H.H.: *Banach Lattices and Positive Operators*, Springer Verlag, Berlin, Heidelberg, New York, 1974. MR0423039 (54 #11023)
5. Scheffold, E.: Über komplexe Banachverbandsalgebren, *J. Functional Analysis* **37** (1980), 382–400. MR0581429 (81m:46064)
6. Scheffold, E.: Banachverbandsalgebren mit einer natürlichen Wedderburn-Zerlegung, *Math. Z.* **185** (1984), 521–531. MR0733772 (85h:46070)
7. Scheffold, E.: Der Bidual von F-Banachverbandsalgebren, *Acta Sci. Math.* **55** (1991), 167–179. MR1124955 (92h:46027)
8. Yosida, K.: *Functional Analysis*, Springer Verlag, Berlin, Heidelberg, New York, 1966. MR0239384 (39 #741)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR1019163 (90j:47046) 47B55 47A10 47A35
Schep, Anton R. **[Schep, Anton Roelof]** (1-SC)
A remark on the uniform zero-two law for positive contractions.
Arch. Math. (Basel) **53** (1989), *no. 5,* 493–496.

Let E be an order complete Banach lattice, and let $T: E \to E$ be a positive contraction. The main result of the paper is the following strong zero-two law for positive contractions of order complete Banach lattices: if $\|T^{m+1} - T^m\|_r < 2$ for some $m \in \mathbf{N} \cup \{0\}$, then $\lim_{n\to\infty} \|T^{n+1} - T^n\|_r = 0$.

The proof uses the order spectrum of order bounded operators defined by H. H. Schaefer [Math. Z. **154** (1977), no. 1, 79–84; MR0470748 (57 #10494)], several simple (but powerful) facts concerning Banach algebras, and previous results obtained by Y. Katznelson and L. Tzafriri [J. Funct. Anal. **68** (1986), no. 3, 313–328; MR0859138 (88e:47006)] and Schaefer [Israel J. Math. **59** (1987), no. 2, 241–244; MR0920086 (88m:47066)]. In proving the main result, the author shows also that $\lim_{n\to\infty} \|T^{n+1} - T^n\|_r = 0$ if and only if the intersection of the order spectrum of T with the unit circle is either empty, or equal to $\{1\}$. The paper also contains several applications of the zero-two law concerning the peripheral order spectrum of a positive contraction, and a semigroup version of the main result.

The zero-two law proved in the paper is a natural extension of the strong zero-two law for positive contractions of L^p-spaces, $1 \le p < \infty$ of R. Wittmann [Math. Z. **197** (1988), no. 2, 223–229; MR0923490 (89d:47017)], while at the same time it strengthens and is more general than the extension of the zero-two law obtained by Katznelson and Tzafriri [op. cit.]. It is the most general strong zero-two law in Banach lattices (the study of the strong zero-two laws was implicitly started by D. S. Ornstein and L. Sucheston, the creators of the first zero-two laws [Ann. Math. Statist. **41** (1970), 1631–1639; MR0272057 (42 #6938)]) obtained so far; its proof is both simple and elegant.

Radu Zaharopol

MR0288557 (44 #5754) 46.06
Schneider, H. [Schneider, Hans]; Turner, R. E. L.
Positive eigenvectors of order-preserving maps.
J. Math. Anal. Appl. **37** 1972 506–515

The authors study positive (not necessarily linear or continuous) maps on a partially ordered Banach space. Their results have applications to the discontinuous Sturm-Liouville problem. They give a new proof of the fact that the spectral radius is in the spectrum of a positive linear operator. *H. E. Lacey*

MR2250317 (2007m:47005) 47A10 15A48 47B65 47H07
Seidman, Thomas I. (1-MD3-MS); **Schneider, Hans** (1-WI)
The spectral radius in partially ordered algebras. (English summary)
Linear Algebra Appl. **417** (2006), *no. 2-3,* 347–369.

Summary: "We prove theorems of Perron-Frobenius type for positive elements in partially ordered topological algebras satisfying certain hypotheses. We show how some of our results relate to known results on Banach algebras. We give examples and state some open questions."
Jor-Ting Chan

References

1. F.L. Bauer, assisted by H. Vogg, M. Meixner, Positivity and Norms, Tech. Univ. München, 1974. MR0418436 (54 #6476)
2. I.M. Gel'fand, Normierte Ringe, Mat. Sbornik N.S.9 51 (1941) 3–24. MR0004726 (3,51f)
3. M.A. Krasnosel'skiĭ, Je.A. Lifshits, A.V. Sobelev, Positive Linear Systems: the Method of Positive Operators, Heldermann Verlag, Berlin, 1989. MR1038527 (91f:47051)
4. W. Rudin, Functional Analysis, second ed., McGraw-Hill, 1991. MR1157815 (92k:46001)
5. H.H. Schaefer, Topological Vector Spaces, Springer, 1970. MR0342978 (49 #7722)
6. T.I. Seidman, H. Schneider, M. Arav, Comparison theorems using general cones for norms of iteration matrices, Linear Algebra Appl. 399 (2005) 169–186. MR2152416 (2006b:65051)
7. Richard S. Varga, Matrix Iterative Analysis, Prentice-Hall, 1962. MR0158502 (28 #1725)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR859735 (88d:47049) 47B55 46B30 47A10
Sourour, A. R. (3-VCTR)

Spectrum-preserving linear maps on the algebra of regular operators.

Aspects of positivity in functional analysis (*Tübingen*, 1985), 255–259, *North-Holland Math. Stud.*, 122, *North-Holland*, *Amsterdam*, 1986.

Let X be a complex Banach lattice and let $L(X)$ denote the algebra of all bounded linear operators on X. An operator T in $L(X)$ is said to be regular if it is a linear combination of positive operators, and the algebra of all regular operators on X is denoted by $L^r(X)$. Given T in $L^r(X)$, denote by $\sigma_0(T)$ and $\sigma(T)$ its spectrum in $L^r(X)$ and $L(X)$, respectively.

The aim of this note is to prove that the following statements (i)–(v) are equivalent for a linear mapping φ from $L^r(X)$ onto $L^r(Y)$, where Y is a second complex Banach lattice: (i) $\sigma(\varphi(T))=\sigma(T)$ for all $T\in L^r(X)$. (ii) $\sigma_0(\varphi(T))=\sigma_0(T)$ for all $T\in L^r(X)$. (iii) φ is a Jordan isomorphism. (iv) φ is either an algebra isomorphism or an algebra anti-isomorphism. (v) φ takes one of the following forms: $\varphi(T)=ATA^{-1}$ $(T\in L^r(X))$ or $\varphi(T)=BT^*B^{-1}$ $(T\in L^r(X))$, where A [resp. B] is a bounded invertible operator from X [resp., from X^*] onto Y.

If φ satisfies (i)–(iv) and, in addition, is order preserving, then the map A [resp., B] in (v) may be chosen to be a lattice isomorphism. As an immediate corollary, the author observes that $L^r(X)$ and $L^r(Y)$ are isomorphic as ordered algebras if and only if X and Y are isomorphic as Banach lattices.

In addition to discussing the proof of the main result, the author indicates how it is related to earlier work concerning spectrum-preserving or invertibility-preserving mappings between Banach spaces or Banach algebras, including a recent paper by him and A. A. Jafarian [J. Funct. Anal. **66** (1986), no. 2, 255–261; MR MR0832991 (87m:47011)]. The paper ends with a number of open questions.

{For the entire collection see MR0859713 (87h:47002)}

T. A. Gillespie

MR502514 (80a:81030) 81C20 46N05
Srinivas, M. D.
Quantum generalization of Kolmogorov entropy.
J. Math. Phys. **19** (1978), *no. 9,* 1952–1961.

The question of generalizing to the quantum realm the notion of Kolmogorov dynamical entropy has been the object of several investigations in recent years; the author cites works by the reviewer [Z. Wahrsch. Verw. Gebiete **29** (1974), 241–252; MR0377536 (51 #13708); Comm. Math. Phys. **49** (1976), no. 3, 191–215; MR0434287 (55 #7254)] and by A. Connes and E. Størmer [Acta Math. **134** (1975), no. 3–4, 289–306; MR0454657 (56 #12906)]. The present proposal differs from these in three major ways. First, rather than using a C^*- or W^*-algebra approach, the author defines here a general dynamical system $(\mathcal{O}, \mu, \varphi_t)$ as an aggregate formed by a quantum event space $\mathcal{O}$, a state μ on $\mathcal{O}$, and a homomorphism φ from $\mathbf{R}$ into Aut $\mathcal{O}$. The event space $\mathcal{O}$, which is to be interpreted as the set of all "operations", is taken to be the set of all positive elements in the unit ball of an ordered Banach algebra with identity; a typical such model is $L_1^+(V) \subset L(V)$, i.e. the set of all bounded linear mappings of a complete base normed space V into itself; in conventional quantum mechanics, $V = T_s(H)$, the space of all selfadjoint, trace-class operators on a separable Hilbert space H. Second, a great emphasis is placed on the fact that the entropy is defined sequentially, as it involves a conjunction $\bigwedge$ defined on $\mathcal{O}$ in such a manner that $A \bigwedge B$ is to be interpreted as the experimental procedure in which the system is subjected to the sequence of experimental procedures A followed by B. Third, no specific example of a dynamical system is given for which the potential usefulness of these concepts could be explicitly exploited. *Gérard G. Emch*

MR0215116 (35 #5959) 47.20
Stecenko, V. Ja.
On a spectral property of an indecomposable operator. (Russian)
Uspehi Mat. Nauk **22** 1967 242–244

The author continues the study of his earlier paper [Uspehi Mat. Nauk **21** (1966), no. 5 (131), 265–267; MR0201977 (34 #1854)]. Under the assumption that A is an indecomposable operator on a Banach lattice and its spectral radius $r(A)$ is an eigenvalue of both A and A^* with positive eigenvectors, $r(A)$ is proved to be a simple eigenvector of A. Another topic is an estimate of the spectral radius of the sum of positive operators $B_1, B_2, \cdots, B_n$ for which $B_iB_j - B_jB_i$ are positive ($i < j, i, j = 1, 2, \cdots, n$). When all B_i are completely continuous, the following estimate is given without proof: $r(\sum_{i=1}^n B_i) \leq \sum_{i=1}^n r(B_i)$. {In the review of the above-mentioned article, "indecomposable" was mistranslated as "irreducible".} *T. Ando*

MR592901 (82b:47048) 47B55 46B30
Synnatzschke, J.
Über einige verbandstheoretische Eigenschaften der Multiplikation von Operatoren in Vektorverbänden. (German)
Math. Nachr. **95** (1980), 273–292.

The author considers complete vector lattices W, X, Y, Z and maps between various associated spaces of regular operators $H_r(X, Y)$, $H_r(W, Z)$. If $A \in H_r(X, Y)$, B is a linear operator from Y to Z and C a linear operator from W to X then BAC is a linear operator from W to Z. Most of the paper is devoted to studying the map ${}_BT_C\colon A \to BAC$ in various guises. Properties such as positivity, regularity, and normality are considered. Components of these maps are also studied, particularly with reference to components of almost integral operators. *S. J. Bernau*

MR0256187 (41 #844) 46.80
Taylor, Joseph L.
Noncommutative convolution measure algebras.
Pacific J. Math. **31** 1969 809–826

From the author's summary: "A convolution measure algebra is a partially ordered Banach algebra in which the norm, order and algebraic operations are related in special ways. Examples include the group algebra $L^1(G)$ and the measure algebra $M(G)$ on a locally compact group G and, more generally, the measure algebra $M(S)$ on any locally compact semigroup S.

"This paper demonstrates several ways in which a convolution measure algebra can be realized as an algebra of measures on a compact semigroup. A relation is established between such realizations and certain classes of Banach space representations of the algebra. These results give a partial extension to the noncommutative case of the structure theory of commutative semi-simple convolution measure algebras." *P. Civin*

MR0336344 (49 #1119) 46H15
Thompson, A. C.; **Vijayakumar, M. S.**
An order-preserving representation theorem for complex Banach algebras and some examples.
Glasgow Math. J. **14** (1973), 128–135.

Let A be a complex Banach algebra with unit e of norm 1. Let $B = \{f \in A^*: f(e) = \|f\| = 1\}$, $M = \{f \in B: f(J) = 0$ for some maximal left ideal J of $A\}$, $\Omega = w^*$-closure of M, S the w^*-closed convex span of M, $C = \bigcup\{\lambda S: \lambda \geq 0\}$, $V = C - iC$ and $K = \{x \in A: \operatorname{Re} f(x) \geq 0$, for all $f \in V\}$. The authors prove the following representation theorem: There is a mapping φ from A into $C(\Omega)$ such that (a) φ is a continuous linear homomorphism from A to a subspace of $C(\Omega)$; (b) φ is an order homomorphism with respect to the order induced by the wedge K and the positive cone P of functions in $C(\Omega)$ with nonnegative real and imaginary parts; (c) φ is an isomorphism if and only if K is a cone; (d) if K is a cone, then φ is a homeomorphism if and only if K is normal. *P. Civin*

MR2373493 (2008m:46042) 46B40 47B65

Toumi, Nedra

On some order ideals in Banach almost f-algebras. (English summary)

Int. Math. Forum **3** (2008), *no. 1-4,* 107–113.

Summary: "Let A be a Banach almost f-algebra, $k \geq 2$ be a natural number and $a_1, \ldots, a_k \in A_+$. Put $b_k = a_1 \cdots a_k$. Then the closure $\overline{b_k A}$ of the set $b_k A$ of all products $b_k a$ with $a \in A$ is a vector lattice under the ordering and the multiplication inherited from A with $\overline{b_k A_+}$ as a positive cone. Moreover, we prove that, for all $a \in A_+$, every principal order ideal I_{a^k} generated by a^k is a subalgebra of A."

MR1080661 (91j:47043) 47B65 47B60
Triki, Abdelmajid (TN-TUNIS)
Extensions of positive projections and averaging operators.
J. Math. Anal. Appl. **153** (1990), *no. 2,* 486–496.

In the first section, it is shown that each positive projection on a majorizing subspace of a Dedekind complete vector lattice E admits an extension to a positive projection on the whole of E. The remaining sections sharpen some results on positive contractive projections and averaging operators due to J. L. Kelley [Illinois J. Math. **2** (1958), 214–223; MR0103409 (21 #2179)] and G. L. Seever [Pacific J. Math. **17** (1966), 159–166; MR0192356 (33 #581)] for $C(K)$-spaces and to C. B. Huijsmans and B. de Pagter [J. Math. Anal. Appl. **113** (1986), no. 1, 163–184; MR0826666 (87f:47055)] in the setting of Archimedean semi-prime f-algebras. *Peter Dodds*

MR2160427 (2006d:46019) 46B42 46A40 46B40
Uyar, Ayşe (TR-GAZI-ED)
On Banach lattice algebras. (English summary)
Turkish J. Math. **29** (2005), *no. 3,* 287–290.

Let E be a Banach lattice f-algebra with unit e, $\|e\| = 1$, in which for every $a \in E^+$ the inverse a^{-1} exists. In this paper, the author shows that E is lattice-and-algebra isometric isomorphic to $\mathbb{R}$ and this without using the assumption $a^{-1} > 0$. She also gives an alternative proof to a result of Huijsmans.

REVISED (February, 2006)

Current version of review. Go to earlier version. *Lahcène Mezrag*

References

1. Aliprantis, C.D. and Burkinshaw, O., Positive Operators, Academic Press, London, 1985 MR0809372 (87h:47086)
2. Beukers, F., Huijsmans, C.B., Pagter, B., Unital embedding and complexification of f-algebras, Math. Z., 183, 131–144, 1983. MR0701362 (85c:06016)
3. Bonsall, F.F. and Duncan, J., Complete normed algebras, Springer, Berlin, 1973. MR0423029 (54 #11013)
4. Huijsmans, C.B. and Pagter, B., Subalgebras and Riesz subspaces of an f-algebra, Proc. Lond. Math. Soc., 48, 3, 161–174, 1984. MR0721777 (85f:46015)
5. Huijsmans, C.B., Elements with unit spectrum in a Banach lattice algebra, Proceedings A, 91,1, 43–51,1988. MR0934473 (89f:46106)
6. Huijsmans, C.B., Lattice-ordered division algebras, Proc. R. Ir Acad. Vol 92 A, 2, 239–241,1992. MR1204223 (94b:06023)
7. Lavric, B., A note on unital Archimedean Riesz algebras, An. Ştiint. Unv. Al. I. Cuza Iaşi Sect I a Mat., 39, 4, 397–400, 1993. MR1328938 (96f:46102)
8. Luxemburg, W.A.J. and Zaanen, A.C., Riesz spaces I, North Holland, Amsterdam, 1971.
9. Schaefer, H.H., Banach lattice and positive operators, Springer, Berlin, 1974. MR0423039 (54 #11023)
10. Zaanen, A.C., Riesz Spaces II, North Holland, Amsterdam, 1983. MR0704021 (86b:46001)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR833906 (87h:46111) 46H99 06F25
Venter, Lucas
A multiplication inequality in complex Banach lattice algebras.
Quaestiones Math. **8** (1985), *no. 3,* 275–281.

Let A be a real lattice-ordered algebra. It is an immediate consequence of the definition that $|ab| \leq |a| \cdot |b|$ for all a, b in A. If A is uniformly complete, then the absolute value in A can be extended to the complexification $A_c = A \oplus iA$ of A by means of the formula $|z| = \sup\{\mathrm{Re}(e^{i\theta} z) \colon 0 \leq \theta \leq 2\pi\}$.

In this paper it is shown that the inequality $|z_1 z_2| \leq |z_1| \cdot |z_2|$ holds for all z_1 and z_2 in A_c. This result contains the special case that A is a Banach lattice algebra [e.g., E. Scheffold, J. Funct. Anal. **37** (1980), no. 3, 382–400; MR0581429 (81m:46064)], and that A is the algebra of order bounded operators on a Dedekind complete vector lattice.

{Reviewer's remark: The result of this paper was also proved by C. B. Huijsmans ["An inequality in complex Riesz algebras", Studia Sci. Math. Hungar., to appear].}

B. de Pagter

MR998747 (90h:46084) 46H99 46A40 47B55
Venter, Lucas (SA-POTCH); **Grobler, Jacobus J.** (SA-POTCH);
van Eldik, Peter (SA-POTCH)
The peripheral spectrum in Banach lattice algebras.
Quaestiones Math. **12** (1989), *no. 2,* 175–185.

This paper is devoted to the spectral properties of positive elements in Banach lattice algebras. In particular, the peripheral spectrum is studied in some detail. Some extensions of the results of E. Scheffold [J. Funct. Anal. **37** (1980), no. 3, 382–400; MR0581429 (81m:46064)] are obtained. The authors make a point of not using any representation theorems in their proofs, and obtain their results by using the intrinsic lattice structure of the Banach algebras involved. *B. de Pagter*

MR886852 (89i:22011) 22D40 22D45 46L55 47B38 47B55 47D10
Vershik, A. M. [Vershik, Anatoliĭ Moiseevich]
Measurable realizations of automorphism groups and integral representations of positive operators. (Russian)
Sibirsk. Mat. Zh. **28** (1987), *no. 1,* i, 52–60.

Introduction (translated from the Russian): "In the 1930s the papers of L. V. Kantorovich laid the foundations of the theory of operators in partially ordered spaces. Positive operators occupied a special place in those papers, and in the contemporary studies of M. G. Kreĭn and S. Kakutani. Although these studies later proceeded in various directions, one or even two interrelated aspects have not been extensively developed. We have in mind the connection with measure theory and ergodic theory, in all their breadth, and questions of operator algebras.

"Positive and regular operators (i.e., the differences of positive operators in the terminology of Kantorovich, B. Z. Vulikh and A. G. Pinsker [*Functional analysis in partially ordered spaces* (Russian), Gos. Izdat. Tekhn.-Teor. Lit., Moscow, 1950; MR0038006 (12,340d)]), in particular, positive contractions in $L^2_\mu(X)$, are the basic object of study in the theory of functional models, ergodic theory, and the theory of Markov processes. Although a great deal is known about positive unitary operators, i.e., automorphisms, the cases of positive contractions and Markov operators (polymorphisms) have been much less studied, and the algebras of operators generated by positive operators or by semigroups of such operators have scarcely been studied at all. At the same time the need for such results exists also in ergodic theory and in the theory of representations, and in the theory of operator algebras. Moreover, the study of the semigroup of Markov positive contractions as a convex compactum is closely connected with difficult and important problems of convex analysis and its applications—a theory in which the outstanding role of Leonid Vital′evich is well known.

"In the present paper, which is dedicated to the memory of Leonid Vital′evich Kantorovich, we consider only a few problems from this field.

"The first of them is related to a problem on measurable realizations of groups of positive unitary operators, i.e., to the question of singular lifting. We show how this well-known problem in ergodic theory, solved earlier by us [Izv. Akad. Nauk SSSR Ser. Mat. **29** (1965), 127–136; MR0172977 (30 #3192)], G. W. Mackey [Illinois J. Math. **6** (1962), 327–335; MR0143874 (26 #1424)] and G. Maruyama [J. Math. Soc. Japan **18** (1966), 303–330; MR0212160 (35 #3035)] can be studied by the general method of linearization, presented in our 1965 paper [op. cit.] for locally compact groups, and we also indicate the complications that arise for infinite-dimensional groups. Then we turn to a question of the integral representation of positive and regular operators. Theorems on the simultaneous integral representation of algebras spanned by positive operators (Section 2) form an analogue of the theorem on the realization of groups. A direct consequence of the theorem in §1 is a theorem on the compatible integral representation of operators of a crossed product that is generated by the action of a group. In §2 we also give an individual integral representation for an arbitrary positive (Markov) contraction. For this purpose we present the basic concepts of the theory of polymorphisms. The concept of a groupoid operator provides an abstract description of positive operators for which our integral representation is just as convenient as it is for operators of a crossed product.

"Questions which are still unsolved relate to nonlocally compact groups and to general polymorphisms. Here (Section 3) we present a number of counterexamples that illustrate the complexity of the problem. In particular, we select a problem on describing a C^*-algebra generated by all positive contractions."

{English translation: Siberian Math. J. **28** (1987), no. 1, 36–43.}

MR859736 (87k:47075) 47B38 47A10
Weis, Lutz (1-LAS)

An extrapolation theorem for the 0-spectrum.

Aspects of positivity in functional analysis (*Tübingen*, 1985), 261–269, *North-Holland Math. Stud.*, 122, *North-Holland*, *Amsterdam*, 1986.

Let (Ω, μ) be a finite measure space and let T be a bounded linear operator from $L_\infty(\Omega, \mu)$ into itself that extends to a bounded linear operator T from $L_p(\Omega, \mu)$ into itself for all $1 \leq p \leq \infty$. Then it is well known that $\sigma(T\big|L_p) \subset \sigma(T\big|L_1) \cup \sigma(T\big|L_\infty)$, where $\sigma(T\big|L_p)$ denotes the spectrum of T as an operator in L_p.

It is shown that if T is a regular operator in $L_p(\mu)$ $(1 < p < \infty)$, then there exists a positive isometry J of $L_p(\mu)$ such that $\tilde{T} = JTJ^{-1}$ extends to an endomorphism $\tilde{T}\colon L_q(\mu) \to L_q(\mu)$ for all $1 \leq q \leq \infty$ and $\sigma_0(\tilde{T}\big|L_q) = \sigma(\tilde{T}\big|L_1) \cup \sigma(\tilde{T}\big|L_\infty)$ for all $1 < q < \infty$, where σ_0 is the order spectrum of $\tilde{T}$ in the sense of H. Schaefer. Similar results are discussed for bounded operators acting on spaces $L_p(X)$, where X is a Banach space.

{For the entire collection see MR0859713 (87h:47002)}

W. A. J. Luxemburg

MR0399866 (53 #3708) 46H99
White, A. J.
Ordered Banach algebras.
J. London Math. Soc. (2) **11** (1975), *no. 2,* 175–178.

A complex algebra A is called an L-algebra if there is a partial order $\leq$ in terms of which A is a complex L-space. If, in addition, A has the property that $a \geq 0$, $b \geq 0$ in A imply $ab \geq 0$ and $\|ab\| = \|a\| \cdot \|b\|$ then A is called an ordered L-algebra. If $F \in A^*$, the set $P_F = \{a \in A \colon F(a) = \|a\|\}$ is a cone in A. F is said to be L-inducing if the partial ordering generated by P_F makes A into a complex L-space.

Suppose that $F \in A^*$ is L-inducing and $\|F\| = 1$. The author shows that A is an ordered L-algebra in the order generated by F if and only if F is a multiplicative linear functional. Suppose that $F \in A^*$ is L-inducing and $a, b \in P_F$ imply $ab \in P_F$; then $|ab| \leq |a| \cdot |b|$ with respect to the order generated by P_F. These results are applied to the theory of a commutative ordered L-algebra B. In particular, they help simplify characterizations of commutative group algebras due to M. A. Rieffel [Trans. Amer. Math. Soc. **116** (1965), 32–65; MR0198141 (33 #6300)] and J. L. Taylor [Acta Math. **126** (1971), 195–225].

B. Yood

MR0420214 (54 #8228) 46A40 47B55

Wickstead, A. W.

The ideal centre of a Banach lattice.

Proc. Roy. Irish Acad. Sect. A **76** (1976), *no. 4*, 15–23.

Let E be a Banach lattice with a topological order unit, and $Z(E)$ the ideal centre of E consisting of all linear operators on E which are bounded in order by some multiple of the identity operator. The author shows that $Z(E)$, under its strong operator topology S, is complete and provides information about E. For example, he shows that E has an order unit if and only if $Z(E)$ is metrizable. Compact operators in $Z(E)$ are S-dense if and only if the order intervals in E are norm-compact. Finally, the order-continuity of the norm on E can also be characterized in terms of properties of $Z(E)$. *Kung-fu Ng*

MR0477671 (57 #17188a) 46A40
Wickstead, A. W.
The structure space of a Banach lattice.
J. Math. Pures Appl. (9) **56** (1977), *no. 1,* 39–54.

MR0477672 (57 #17188b) 46A40
Wickstead, A. W.
The structure space of a Banach lattice. II.
Proc. Roy. Irish Acad. Sect. A **77** (1977), *no. 9,* 105–111.

In the first part of this paper the author gives a representation of the ideal centre of a semisimple Banach lattice E. Let E be a Banach lattice; then the ideal centre $Z(E)$ of E is the space of all linear operators T on E for which there exists $n \in N$ with $-nx \leq Tx \leq nx$ for all $x \in E_+$. The author calls E semisimple if every closed ideal I in E, such that $I + J = E$ for some closed ideal J, is equal to $\{0\}$. Let $F(E)$ be the set of all weak*-closed faces of E^*_+. Then $M(E)$ will denote the set of all families of elements of $F(E)$ which have the property that finite intersections of members are nonzero, and which are maximal with respect to this property. $M(E)$ is called the structure space of B. If $G \in F(E)$ let $G^\vee = \{A \in M(E) \colon G \in A\}$. As G varies over $F(E)$ the sets $G^\vee$ form a base for the closed sets of a compact T_1 topology on $M(E)$.

Then the main result of this paper is the following theorem: Let E be a semisimple Banach lattice; there is an isometric order and algebra isomorphism $T \to T^\pi$ of $Z(E)$ onto $C(M(E))$ such that, for $F \in F(E)$ and $T \in Z(E)$, $T^*|_F \geq 0$ if and only if $T^\pi|_{F^\vee} \geq 0$.

In the second part the author studies the relationship between the structure space of two Banach lattices E and F when there is a linear lattice homomorphism $T \colon E \to F$. A typical example of the results in this direction is the following: Let E be a simple Banach lattice (i.e., all its quotients by closed ideals are semisimple), I a closed ideal in E and such that both E and J have Hausdorff structure spaces. Then $M(J)$ is homeomorphic to the Stone-Čech compactification of $M(E) \setminus (J^0_+)^\vee$. *N. Popa*

MR620620 (82g:47027) 47B55 46A40 47D20
Wickstead, A. W.
Relatively central operators on Archimedean vector lattices. I.
Proc. Roy. Irish Acad. Sect. A **80** (1980), *no. 2,* 191–208.

Let E be an Archimedean vector lattice (Riesz space) and H a vector sublattice (Riesz subspace). The linear operator $T\colon E\to H$ is said to be central relative to H if there exists a number $\lambda\ge 0$ such that $x\in E$, $h\in H$ and $|x|\le|h|$ together imply that $|Tx|\le\lambda|h|$. This restricts the action of T only on the ideal generated by H. Therefore it is assumed in the greater part of the paper that H is cofinal. The space of all linear operators from E into H that are central relative to H is denoted by $Z(E|H)$. If $E=C(X)$ with X compact and Hausdorff and H is closed and contains all constants, then $Z(E|H)$ consists precisely of all averaging operators, i.e., $T(fh)=(Tf)\cdot h$ for all $f\in C(X)$, $h\in H$. For the operator order, $Z(E|H)$ need not be a lattice, but it is a lattice if H is Dedekind complete. If E is a Banach lattice and H is norm closed (and cofinal), the compact operators in $Z(E|H)$ form a lattice. There exists a natural norm $\|T\|_n$ on $Z(E|H)$, defined by

$$\|T\|_n=\inf\{\lambda>0\colon |Tx|\le\lambda|h| \text{ for } x\in E, h\in H \text{ and } |x|\le|h|\}.$$

If E is normal, $\|T\|_n$ and the operator norm need not be equivalent, but they are if E is a Banach lattice. For general E and H Dedekind complete, the space of all order continuous operators in $Z(E|H)$ is an injective Banach lattice. *A. C. Zaanen*

MR1125740 (93g:47048) 47B65 46B42 47D30
Wickstead, A. W. (4-QUEEN)
An embedding of the algebra of order bounded operators on a Dedekind complete Banach lattice.
Math. Z. **208** (1991), *no. 1,* 161–166.

Let E be a Banach lattice; $\mathcal{L}^{\mathrm{r}}(E)$ is the space of regular operators on E, i.e., the set of differences of operators ≥ 0, and $\mathcal{L}^{\mathrm{r}}(E)$ is complete for the norm $\|T\|_{\mathrm{r}} = \sup\{\|\sup T([-x,x])\|;\ x \leq 1\}$. Theorems: Assume moreover that E is Dedekind complete in (a), (b), (c). (a) If $T \geq 0$ and is sup-preserving then $\sigma_0(T)$ is cyclic ($\sigma_0(T)$ denotes the spectrum for $\|\ \|_{\mathrm{r}}$), i.e., $(re^{i\theta} \in \sigma_0(T)) \Rightarrow (re^{in\theta} \in \sigma_0(T)$ for all n). (b) If $T \geq 0$ and transforms intervals into intervals then $\sigma_0(T)$ is cyclic. (c) If $T \geq 0$ and is G-solvable then the subset of $\sigma_0(T)$ of maximal diameter is cyclic. (d) If $T \geq 0$ and transforms intervals into intervals then $\sigma(T)$ is cyclic.

The method consists of associating with E and with a set I of cardinality $>|E|$ the space $\widehat{E}$ of mappings f of I into E with values in a (nonfixed) interval of E; the author puts $\|f\| = \inf\{\|x\|\colon |f(i)| \leq x$ for all $i \in I\}$. *Richard Becker*

MR1930989 (2004b:47069) 47B65 47B07 47L05
Wickstead, A. W. (4-QUEEN-PM)
The centre of spaces of regular operators.
Math. Z. **241** (2002), *no. 1,* 165–179.

This interesting paper, in a way, has its origin in [G. J. H. M. Buskes et al., Nederl. Akad. Wetensch. Indag. Math. **48** (1986), no. 1, 1–9; MR0834315 (87f:47052)]. In the latter an embedding of the algebraic tensor product of the center of two vector lattices E and F, denoted $Z(E) \odot Z(F)$, into the center of the regular operators from E to F, denoted $Z(\mathcal{L}^{\mathrm{r}}(E,F))$, was used to obtain an up-down theorem in $\mathcal{L}^{\mathrm{r}}(E,F)$. The present paper considers the injective tensor norm on $Z(E) \odot Z(F)$ and the order unit norm on $Z(\mathcal{L}^{\mathrm{r}}(E,F))$ and then shows that the embedding into $Z(\mathcal{L}^{\mathrm{r}}(E,F))$ preserves the norm if E and F are uniformly complete and the order dual of E separates the points. Thus the embedding extends to an isometry (as well as algebra and order isomorphism) of the injective tensor product $Z(E) \odot_\lambda Z(F)$ into $Z(\mathcal{L}^{\mathrm{r}}(E,F))$. The author then further focuses on density of the embedding with applications to weakly compact and compact operators. As just one illustration out of many of his results, he proves a theorem that characterizes exactly the Banach lattices E, F that have the property that if $0 \le S \le T$ are operators $E \to F$ and T is r-compact [see W. Arendt, Math. Z. **178** (1981), no. 2, 271–287; MR0631633 (83h:47027)], then S is r-compact. For further details and results we refer the reader to the paper under review. *Gerard Buskes*

References

1. Y. A. Abramovich, L. P. Janovsky, Applications of the Rademacher systems to operator characterizations of Banach lattices, Colloq. Math. **46** (1982), 75–78 MR0672365 (84h:46029)
2. C. D. Aliprantis, O. Burkinshaw, The components of a positive operator, Math. Z. **184** (1983), 245–257 MR0716274 (85b:47040)
3. C. D. Aliprantis, O. Burkinshaw, Positive Operators, Academic Press, New York & London, 1985 MR0809372 (87h:47086)
4. W. Arendt, On the o-spectrum of regular operators and the spectrum of measures, Math. Z. **178** (1981), 271–287 MR0631633 (83h:47027)
5. W. Arendt, A.R. Sourour, Perturbation of regular operators and the order essential spectrum, Indag. Math. **49** (1986), 109–122 MR0849713 (87j:47052)
6. R.C. Buck, Multiplication operators, Pacific J. Math. **11** (1961), 95–104 MR0123184 (23 #A513)
7. G.J.H.M. Buskes, P.G. Dodds, B. de Pagter, A.R. Schep, Up-down theorems in the centre of $\mathcal{L}_b(E,F)$, Indag. Math. **49** (1986), 1–9 MR0834315 (87f:47052)
8. D.I. Cartwright, H.P. Lotz, Some characterizations of AM- and AL-spaces, Math. Z. **142** (1975), 97–103 MR0383030 (52 #3912)
9. Z.L. Chen, A.W. Wickstead, Incompleteness of the linear span of the positive compact operators, Proc. Amer. Math. Soc., (to appear) MR1443816 (98k:47072)
10. P.G. Dodds, D.H. Fremlin, Compact operators in Banach lattices, Israel J. Math. **34** (1979), 287–320 MR0570888 (81g:47037)
11. Goullet de Rugy, La structure idéale des M-espaces, J. Math. pures et appl. **51** (1972), 331–373 MR0385512 (52 #6373)
12. B. Jefferies, S. Okada, An operator bound related to regular operators, Arch. Math.

66 (1996), 219–227 MR1373249 (97a:47042)
13. P. Meyer-Nieberg, Banach Lattices, Springer-Verlag, Berlin Heidelberg New York, 1991 MR1128093 (93f:46025)
14. B. de Pagter, The components of a positive operator, Indag. Math. **46** (1983), 229–241 MR0705429 (85c:47036)
15. H. Raubenheimer, The o-spectrum of r-asymptotically quasi-finite-rank operators, Quaest. Math. **7** (1984) 299–303 MR0771655 (86c:47050)
16. H. Raubenheimer, r-asymptotically quasi-finite-rank operators and the spectrum of measures, Quaest. Math. **10** (1986) 97–111 MR0857772 (87m:47090)
17. H.U. Schwarz, Banach lattices of bounded operators, Math. Nachr. **90** (1979), 103–108 MR0550990 (81b:47054)
18. A. W. Wickstead, The ideal centre of a Banach lattice, Proc. Royal Irish Acad. **76** (1976), 15–23 MR0420214 (54 #8228)
19. A. W. Wickstead, Extremal structure of cones of operators, Quart. J. Math. Oxford **32** (1981), 239–253 MR0615198 (82i:47069)
20. A. W. Wickstead, Banach lattices with trivial centre, Proc. Royal Irish Acad. **88** (1988), 71–83 MR0974286 (89m:46030)
21. A. W. Wickstead, Dedekind completeness of some lattices of compact operators, Bull. Pol. Acad. Sci. **43** (1995), 297–304 MR1414786 (97k:47032)
22. A. W. Wickstead, Converses for the Dodds-Fremlin and Kalton-Saab theorems, Math. Proc. Camb. Phil. Soc. **120** (1996), 175–179 MR1373356 (96m:47067)
23. W. Wils, The ideal centre of partially ordered vector spaces, Acta Math. **127** (1971), 41–77 MR0463881 (57 #3819)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2179776 (2006e:46056) 46H35 46B42 47B49 47B65 47L10 47L45
Wickstead, A. W. (4-QUEEN-PM)
Order and algebra isomorphisms of spaces of regular operators. (English summary)
Math. Ann. **332** (2005), *no. 4,* 767–774.

Let X and Y be (real) Banach spaces. It is well known that if $\mathcal{U}\colon \mathcal{L}(X) \to \mathcal{L}(Y)$ is an algebra isomorphism then there exists a bounded bijective linear operator $U\colon X \to Y$ such that $\mathcal{U}(T) = UTU^{-1}$ for all $T \in \mathcal{L}(X)$ [C. E. Rickart, *General theory of Banach algebras*, D. van Nostrand Co., Inc., Princeton, N.J., 1960; MR0115101 (22 #5903)]. In the paper under review, the author provides an interesting Banach lattice version of this classical result. More precisely, he proves that if X and Y in addition are Banach lattices and if $\mathcal{U}\colon \mathcal{L}^r(X) \to \mathcal{L}^r(Y)$ is a lattice-ordered algebra isomorphism, then there is a lattice isomorphism $U\colon X \to Y$, so that U and U^{-1} are automatically bounded, such that $\mathcal{U}(T) = UTU^{-1}$ for all $T \in \mathcal{L}_r(X)$. Here $\mathcal{L}_r(X)$ denotes the lattice-ordered algebra of all regular linear operators on X. Moreover, the author shows that each of the extra conditions '$\mathcal{U}$ is an isometry for the operator norm' and '$\mathcal{U}$ is an isometry for the regular norm' turns out to be necessary and sufficient for the associate linear operator U in order to be isometric. *Karim M. Boulabiar*

References

1. Fremlin, D.H.: Tensor Products of Archimedean Vector Lattices. Amer. J. Math. **94**, 777–798 (1972) MR0312203 (47 #765)
2. Fremlin, D.H.: Tensor Products of Banach Lattices. Math. Ann. **211**, 87–106 (1974) MR0367620 (51 #3862)
3. Fremlin, D.H., Talagrand, M.: A Decomposition Theorem in Tensor Products of Archimedean Vector Lattices. Mathematika **26** 302–305 (1979) MR0575648 (81i:46026)
4. Meyer-Nieberg, P.: Banach Lattices. Springer-Verlag, Berlin-Heidelberg, 1991 MR1128093 (93f:46025)
5. Rickart, C.E.: General Theory of Banach Algebras. Van Nostrand, Princeton, 1960 MR0115101 (22 #5903)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2529410 (2010h:46024) 46B42 47B60 47L10
Wickstead, A. W. (4-QUEEN-CPM)
Banach lattices with topologically full centre. (English, Russian summaries)
Vladikavkaz. Mat. Zh. **11** (2009), *no. 2, front matter*, 50–60.

The centre $Z(E)$ of a vector lattice E consists of those linear operators on E which satisfy the inequality $-\lambda x \le Tx \le \lambda x$ for all $x \in E_+$ and some $\lambda \in \mathbb{R}_+$. The centre $Z(E)$ of a Banach lattice E is said to be topologically full if, whenever $0 \le x \le y$, $x, y \in E$, there is a sequence (T_n) in $Z(E)$ such that $T_n(y) \to x$. Banach lattices with topologically full centres were introduced by the author in [Quart. J. Math. Oxford Ser. (2) **32** (1981), no. 126, 239–253; MR0615198 (82i:47069)]. "When is the centre a maximal abelian subalgebra of continuous operators $L(E)$ on E?" is the first question the author deals with. In a Dedekind σ-complete Banach lattice, $Z(E)$ is a maximal abelian subalgebra of $L(E)$. In Theorem 2.4, the author gives a simple proof to show that if E is a Banach lattice with topologically full centre, then $Z(E)$ is a maximal abelian subalgebra of $L(E)$. In 1988 M. Orhon [J. Karadeniz Tech. Univ. Fac. Arts Sci. Ser. Math.-Phys. **11** (1988), 21–32 (1989); MR1103157 (92b:47050)] asked: if $Z(E)$ is a maximal abelian subalgebra of $L(E)$, then must $Z(E)$ be topologically full? The author presents an example of an AM-space H with centre $Z(H)$ not topologically full, but so that $Z(E)$ is a maximal abelian subalgebra of $L(H)$. The author defines a linear subspace J of a linear space E to be algebraically hyperinvariant for a linear operator T on E if J is invariant for all linear operators on E which commute with T. In Theorem 3.1, it is shown that if E is an Archimedean vector lattice and $T \in Z(E)$ which is not a scalar multiple of the identity then there is a proper algebraically hyperinvariant band for T. It is also shown that if E is a Banach lattice with topologically full centre and $T \in Z(E)$ and J is a norm closed hyperinvariant subspace for T, then J is an order ideal. The paper contains interesting conjectures and questions. *Şafak Ömer Alpay*

MR0463881 (57 #3819) 46A40
Wils, Wilbert
The ideal center of partially ordered vector spaces.
Acta Math. **127** (1971), 41–77.

This paper is concerned with a partially ordered vector space E over R such that $E = E^+ - E^+$. For the most part it is assumed that E is the dual of a Banach space A with a partial ordering that is regular in the sense of Davies, and such that the open unit ball of A is directed. This implies that the ordering of E is regular and that the norm is additive on E^+. The positive part of the closed unit ball of E, denoted by K, is then the prototype of a compact convex set.

Reinventing a device of R. C. Buck [Pacific J. Math. **11** (1961), 95–103; MR0123184 (23 #A513)] the author defines the ideal center of E as the set Z_E of order bounded endomorphisms of E. He then shows that Z_E is an order-complete commutative algebra and that the extremal points of the positive part of the closed unit ball of Z_E, identified with the idempotent elements, form a boolean algebra, closed under arbitrary intersections. Then a set G in E^+ is a split face in the sense of Alfsen and Andersen if and only if $G = TE^+$, where T is an idempotent in Z_E. With Δ denoting the set of extremal points of K, equipped with the facial topology, a simple proof of the isomorphism between $C(\Delta)$ and the weak* continuous elements of Z_E (identified with $Z_A{}^*$) is given.

The most important part of the paper concerns the construction, for each g in K, of a probability measure μ_g on K with barycenter g and pseudo-concentrated on the primary elements of K (an element a of E^+ is primary if the ideal center Z_a of the order ideal V_a of E generated by a is trivial). It is proved that such a "central" measure exists and that it is the unique measure representing g that maps $L^\infty(K, \mu_g)$ isomorphically onto $Z_g g$ under the map $\varphi \mapsto \int \varphi g \, d\mu_g$. This result generalizes the unique boundary measure formula for simplices to arbitrary compact convex sets since the primary points of a simplex coincide with the extreme points (in general they form a larger set). It also generalizes the result of S. Sakai and the author from C^*-algebra theory, that each state can be canonically disintegrated in primary states (i.e., states that induce factor representations of the C^*-algebra). In view of the importance of disintegration theory for operator algebras, the concept of central measures may well prove the most important tool in convexity theory since the Choquet-Bishop-de Leeuw development.

One complication in this general setting is the fact that a split face of an order ideal of E need not arise from a split face of E^+, so that for example Z_g cannot be regarded as a quotient of Z_E. This makes the identification of the primary points rather difficult, and indeed it is an open problem whether the primary points form a Borel subset of K, in the case when E is metrizable. The author presents a condition (Ext), trivially satisfied for simplices and for state spaces of C^*-algebras, that assures the surjectivity of the homomorphism $Z_E \to Z_g$ for each g in E^+.

In § 2 of the paper the author claims a number of theorems for regular spaces. But in this case the set of extremal rays of E^+ may well be void (take $E = L^\infty(R, \lambda)$ with $\lambda =$ Lebesgue measure), and so the contents of 2.5, 2.7 and 2.8 are wrong as stated. However, if one assumes the norm to be additive on E^+, the theorems are all restored.

{This review was received in 1971.} *G. K. Pedersen*

MR669926 (84f:47048) 47D10 22B99
Wolff, Manfred

Group actions on Banach lattices and applications to dynamical systems.
Toeplitz centennial (Tel Aviv, 1981), pp. 501–524, *Operator Theory*: *Adv. Appl.*, 4, *Birkhäuser*, *Basel-Boston*, *Mass.*, 1982.

Let G be a locally compact abelian group and U an action of G on a Banach space X. If U is nonquasianalytic, then there is a fairly good theory of spectral subspaces and spectrum for U [see Yu. I. Lyubich et al., Funktsional. Anal. i Prilozhen. **7** (1973), 52–61; MR0348036 (50 #534); Y. Domar and L.-Å. Lindhal, Ann. Inst. Fourier (Grenoble) **25** (1975), 1–32; MR0399872 (53 #3714); I. Ciorănescu and the reviewer, Rev. Roumaine Math. Pures Appl. **21** (1976), 817–850; MR0417856 (54 #5904)]. Assuming that X is a Banach lattice, the $U(g)$'s lattice isomorphisms and U nonquasianalytic, the author deals with conditions under which the spectrum $\sigma(U)$ of U is the whole dual group $G\hat{}$ of G. Sufficient conditions are (up to mild additional restrictions): U nondegenerate (for each compact $0 \notin K \subset G$ there is $0 < x \in X$ with $\inf\{U(g)x, x\} = 0$ for all $g \in K$) or U ergodic and injective. If $G\hat{}$ is "rich" ($\mathbf{R}^n$, $\mathbf{T}^n$ and $\mathbf{Z}$ are rich, but $\mathbf{Z}^n$ is, for $n \geq 2$, poor), then U nondegenerate is also a necessary condition for $\sigma(U) = G\hat{}$ (up to an additional restriction as above).

{For the entire collection see MR0669898 (83h:47002)}

László Zsidó

MR770702 (86e:46046) 46J25 03H05 22D10
Wolff, Manfred P. H. (D-TBNG)

Spectral theory of group representations and their nonstandard hull.

Israel J. Math. **48** (1984), *no. 2-3,* 205–224.

This is a study of the spectrum of a strongly continuous representation of a locally compact abelian group on a Banach space. In particular, results are obtained on the Riesz part of the spectrum and on representations on Banach lattices. Some ideas go back to Yu. I. Lyubich [Dokl. Akad. Nauk SSSR **200** (1971), 777–780; MR0288215 (44 #5413)]. A basic tool in the paper is an extension of the representation within the framework of nonstandard analysis. *Y. Domar*

MR849596 (87j:46045) 46B30 46J10 54H99
Zapata, Guido (BR-FRJ)

The Stone-Weierstrass theorem and a class of Banach lattice algebras.

Aspects of mathematics and its applications, 913–942, *North-Holland Math. Library*, 34, *North-Holland*, *Amsterdam*, 1986.

Fixing a locally compact Hausdorff space X, let $C_0(X)$ denote the space of all real-valued continuous functions on X which vanish at infinity, and consider a subset A of $C_0(X)$ for which the following properties hold: (i) A is both a vector sublattice and a subalgebra of $C_0(X)$; (ii) A strongly separates the points of X; (iii) A is equipped with a lattice norm under which it is complete. The Banach lattice algebras of this type which have the Stone-Weierstrass property (i.e., any subalgebra which strongly separates the points of X is necessarily dense) are identified as those for which the lattice norm is σ-continuous. Moreover, this "Dini condition" is shown to be equivalent to various other classical assertions describing closures of algebraic entities such as ideals or vector sublattices. An alternate characterization of the Banach lattice algebras with σ-continuous norm that occur in the setting at hand provides perspective on these results.

{Reviewer's remark: The author is in good company when it comes to putting a "norm" on the complexification of a real Banach space (p. 928), but this oversight probably does not affect the validity of his work in that direction.}

{For the entire collection see MR0849544 (87f:00014)}

W. H. Summers

MR1184885 (93j:47056) 47B65 47A10
Zhang, Xiao-Dong [**Zhang, Xiao Dong**[3]] (1-FLAT)
Some aspects of the spectral theory of positive operators. (English summary)
Positive operators and semigroups on Banach lattices (Curaçao, 1990).
Acta Appl. Math. **27** (1992), *no. 1-2,* 135–142.

The author considers various aspects of the following (still open) problem: Let T be a positive linear operator on a Banach lattice such that $\sigma(T) = \{1\}$. Is it true that $T \geq I$? The first main result of the paper is that the answer is affirmative under the additional hypothesis that there exist $0 < \alpha < \frac{1}{2}$ and a constant $c \geq 0$ such that $\|T^{-n}\| = O(\exp(cn^{\alpha}))$ as $n \to \infty$. In particular, if 1 is a pole of the resolvent of T, then the answer is affirmative. The author also discusses the related question: Let T be a positive contraction operator on a Banach lattice such that $\sigma(T) \subset \{z : |z| = 1\}$ properly. Is it true that T is an isometry? The author shows that the answer is affirmative in case T^{-n} satisfies the same growth condition as above or when T is a lattice homomorphism.

{For the entire collection see MR1184871 (93e:47003)}

Anton Schep

MR1213328 (94b:47047) 47B65 46A40 47A10 47B60
Zhang, Xiao-Dong [**Zhang, Xiao Dong**[3]] (1-FLAT)
On spectral properties of positive operators. **(English summary)**
Indag. Math. (N.S.) **4** (1993), *no. 1,* 111–127.

The peripheral spectrum of an operator $T: X \to X$ on a Banach space is the nonempty compact set $\mathrm{Per}(T) = \{\lambda \in \sigma(T): |\lambda| = r(T)\}$, where $r(T)$ denotes the spectral radius of T. The peripheral spectrum of T is said to be cyclic if $r(T)e^{i\theta} \in \mathrm{Per}(T)$ implies $r(T)e^{in\theta} \in \mathrm{Per}(T)$ for each integer n. The classical Perron-Frobenius theorem asserts that the peripheral spectrum of a nonnegative matrix is cyclic (as a matter of fact it consists of kth roots of unity for some k). The long-standing problem for positive operators on Banach lattices is whether or not their peripheral spectrum is cyclic.

In this work the author studies in a systematic manner some spectral properties of positive operators that allow him to obtain (among other things) several interesting cases of positive operators with cyclic peripheral spectrum. In particular, the author considers an invertible positive operator T on a complex Banach lattice E with $r(T) = 1$ and such that the unbounded connected component of $\rho(T)$ contains zero. Here is a list of some sample results obtained by the author for such positive operators T. (In what follows I denotes the identity operator on E.) (1) There exist a positive number a and a positive integer k such that $T^k \geq aI$. (2) If T is also a lattice isomorphism satisfying $\sigma(T) = \{1\}$, then $T = I$. (3) There exists a positive integer k such that $\mathrm{Per}(T^k) = \{1\}$. (4) If $\sigma(T)$ is contained in the unit circle, then either $\sigma(T)$ coincides with the unit circle or else $\sigma(T)$ is cyclic and consists of kth roots of unity for some positive integer k. (5) If E is finite-dimensional and $\sigma(T) = \{1\}$, then $T \geq I$. If, in addition, T is a contraction, then $T = I$.

Several examples illustrate the delicate points of the theorems. For more results and proofs, we refer the reader to the interesting paper. *C. D. Aliprantis*

MR1746709 (2001m:47011) 47A11 34K40 47B99
Zima, Mirosława (PL-PURZ)
On the local spectral radius in partially ordered Banach spaces.
Czechoslovak Math. J. **49(124)** (1999), *no. 4,* 835–841.

Let A be a bounded linear operator on a Banach space X. If $x \in X$, then the local spectral radius of A at x is $r(A,x) := \limsup_{n\to\infty} \|A^n x\|^{1/n}$. If operators A and B commute, then it is known that $(*)$ $r(A+B,x) \le r(A,x) + r(B)$ and $r(AB,x) \le r(A,x)\,r(B)$ for each $x \in X$ [J. Daneš, Časopis Pěst. Mat. **112** (1987), no. 2, 177–187; MR0897643 (88j:47004)]. In this paper, the author gives alternative conditions that suffice for $(*)$ and applies them in the setting of functional-differential equations of neutral type.

Thomas Len Miller