Primes of the form $x^2 + ny^2$

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In one of his letters Fermat posed a statement on odd prime numbers saying that $p = x^2 + y^2$ where x and y are integers if and only if $p \equiv 1 \mod 4$. He also claimed to have proven similar results for primes of the form $x^2 + 2y^2$ and $x^2 + 3y^2$. However no conclusive proof was given by him. It was Euler who managed to give a full proof on those results, but he also left some questions open regarding primes of the form $x^2 + 5y^2$ and $x^2 + 14y^2$. It was the theory of quadratic forms developed by Lagrange and Legendre that managed to answer those.

In this talk the proofs of Euler will be given and quadratic forms will be introduced to give further results. If time allows, we will also cover the first part of genus theory to answer questions like for which primes p, q we have that $pq = x^2 + 5y^2$.