Primes and Knots

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The challenge in this project is to explore the following analogy:
$\mathbb{Z}$ is like $\mathbb{R}^3$ and a prime number is like a knotted circle inside $\mathbb{R}^3$.

For example the way two knotted circles p, q are linked in $\mathbb{R}^3$ can be described using the linking number $Lk(p, q)$. Roughly speaking it counts number of times p pierces through any surface bounded by q. The analogous construction for prime numbers p, q is the Legendre symbol $\left(\frac{p}{q}\right)$ describing whether or not p is a square modulo q. When $p, q = 1 \mod 4$ quadratic reciprocity says $\left(\frac{p}{q}\right) = \left(\frac{q}{p}\right)$. The topological equivalent is $Lk(p, q) = Lk(q, p)$.

What about primes in rings of integers of another number field? And how about other properties of knots such as the Alexander polynomial? To make sense of all this we study a topological version of Galois theory and an arithmetic incarnation of the fundamental group. Much more can and should be said, some of which may be found in the recent book [1].

References

[1] M. Morishita *Knots and Primes*, Springer 2012.