Titles and Abstracts

Workshop 'Fractals, Ergodic Theory and Number Expansions'

If the times and days mentioned below do not correspond with the schedule, then the schedule is correct.

Henk Bruin (Universität Wien): Wed 9:30 – 10:30

Title: Matching in generalized beta-transformations

Abstract: The phenomenon of matching (i.e., merging of left/right limits at a discontinuity after some iterates) has been observed in many families with, in some sense, nice algebraic properties. It turn out that also in the family $x \mapsto \beta x + a \mod 1$, matching occurs for many values of β. In this talk I would like to describe the mechanisms behind this. This is based on joint work with Carlo Carminati and Charlene Kalle.

Carlo Carminati (Università di Pisa): Tue 15:45 – 16:30

Title: Regularity and bifurcation phenomena in simple families of piecewise affine maps

Abstract: We study the mechanism which leads to stability and bifurcation phenomena in a simple family of piecewise affine maps. This phenomenon is apparent from the self similar structure of the entropy function. In the case examined we detected also a phenomenon analogous to the tuning for the logistic family. (joint work with Henk Bruin, Stefano Marmi and Alessandro Profeti).

Juan Deng (Shenzhen University): Fri 12:00 – 12:45

Title: C^1-Embeddings between graph-directed sets

Abstract: For graph-directed sets, we obtain that a C^1-embedding implies an affine embedding. We only pose the open set condition for the image sets. The result can be applied to similar sets with overlaps, for example all λ-Cantor sets.

Kan Jiang (Universiteit Utrecht): Thu 15:45 – 16:30

Title: Partition the self-similar sets by periodic orbits

Abstract: In this talk I introduce a very simple idea which enables us to partition some self-similar sets in terms of periodic orbits. Our idea has many potential applications for various problems. For instance, to calculate the Hausdorff dimension (measure) of the set of points in the attractor with exactly k different codings, to give a necessary and sufficient condition under which the self-similar measure is absolutely continuous with respect to the Hausdorff measure, to calculate the exact packing measure for some overlapping self-similar sets. This is joint work with Karma Dajani, and it is the central idea of my thesis.

Vilmos Komornik (Université de Strasbourg): Tue 9:30 – 10:30

Title: Cantor type functions in non-integer bases

Abstract: Cantor's ternary function is generalized to arbitrary base-change functions in non-integer bases. Some of them share the curious properties of Cantor's function, while others behave quite differently. This is a joint work with C. Baiocchi and P. Loreti.

Derong Kong (Universiteit Leiden): Tue 11:00 – 12:00

Title: Entropy, Topological transitivity and Dimensional properties of unique q-expansions

Abstract: Let M be a positive integer and $q \in (1, M+1]$. We consider expansions of real numbers in base q over the alphabet $\{0, \ldots, M\}$. In particular, we study the set $\mathcal{U}_q$ of real numbers with a unique q-expansion, and the set $\mathcal{U}_q'$ of corresponding sequences. Komornik et al (2016) showed that the function $H(q) = h(\mathcal{U}_q')$ is a devil's staircase, where $h(\mathcal{U}_q')$ denotes the topological entropy of $\mathcal{U}_q'$.

In this talk we explicitly determine the plateaus of $H(q)$, and characterize the set $\mathcal{E}$ of non-differential points of $H(q)$. Moreover, we show that the set $\mathcal{E}$ is a Cantor set of full Hausdorff dimension. During our proof we investigate the topological transitivity of a naturally occurring subshift $\mathcal{V}_q'$ related to $\mathcal{U}_q'$. In particular, the subshift $\mathcal{V}_q'$ has a close connection with the open dynamical systems. Finally, we prove that the Hausdorff and box dimensions of $\mathcal{U}_q$ coincide for any $q \in (1, M+1]$.

This is a joint work with Rafael Alcaraz Barrera and Simon Baker.

Wenxia Li (East China Normal University): Thu 9:30 – 10:30

Title: Hausdorff dimensions of sets related with the Lüroth expansions

Abstract: We consider the set of real numbers in $[0, 1]$ which have the prescribed group frequencies of digits in their Lüroth expansions. It is proved that the Hausdorff dimension of such a set is equal to the supremum of the Hausdorff dimensions of its subsets with prescribed digit frequencies in their Lüroth expansions.

Milton Minervino (I2M, Aix-Marseille University): Wed 14:30 – 15:15

Title: Rauzy fractal trees

Abstract: The Rauzy fractal is a geometric representation of a subshift generated by a primitive Pisot substitution. We give an algorithm (based on the dual substitution) to construct a self-similar tree which approximates the Rauzy fractal. This allows to link the dynamics of the subshift with an interval exchange transformation.

Nikita Sidorov (University of Manchester): Wed 15:45 – 16:30

Title: Self-affine sets: topology and arithmetic

Abstract: Let M be a non-degenerate real contracting $d \times d$ matrix and let the IFS given by the maps $M\mathbf{x} + \mathbf{v}_j$, $j = 1, ..., m$, where the $\mathbf{v}_j$ are some translation vectors.

In my talk I will discuss topological and arithmetic properties of its attractor A_M , such as:

- Is it connected or totally disconnected?
- Does it have non-empty interior?
- Which points of A_M have a unique address?

I will also talk about links with simultaneous β-expansions.

This talk is based on my three recent papers with Kevin Hare (Waterloo).

Jörg Thuswaldner (Montanuniversität Leoben): Fri 9:30 – 10:30

Title: S-adic words, Rauzy fractals, and torus rotations

Abstract: In the 1970ies G.Rauzy devised a proof of the fact that Sturmian words are natural codings of rotations on the 1-dimensional torus using classical continued fraction expansions. In an attempt to generalize this to higher dimensions, Arnoux and Rauzy (1991) invented sequences over three letters, now called Arnoux-Rauzy words. They conjectured that each of these words codes a rotation on the 2-torus. In the meantime, this conjecture only could be confirmed on some "periodic" examples. Moreover, pathological counterexamples have been given by Cassaigne, Ferenczi, and Zamboni (2000).

We set up a general theory for the geometry of S-adic sequences that leads to a proof of the conjecture of Arnoux and Rauzy for almost all Arnoux-Rauzy words (w.r.t. a natural measure). Besides that we give non-periodic Arnoux-Rauzy words fulfilling the conjecture. Moreover, we give examples for our new theory that correspond to S-adic words defined in terms of Brun's continued fraction algorithm.

This is joint work with V. Berthé and W. Steiner

Sascha Troscheit (University of St. Andrews): Thu 14:30 – 15:15

Title: Differentiability of distribution functions

Abstract: Given a Gibbs measure supported on a self-conformal set we consider its distribution function. By Lebesgue's theorem any continuous non-decreasing function must be differentiable almost everywhere, but we present more detailed information of the size (in terms of Hausdorff and packing dimension) of the set of points which do not have a derivative.

Jimmy Tseng (University of Bristol): Tue 14:30 – 15:15

Title: Bounded Lüroth expansions and related fractals coming from ergodic dynamical systems

Abstract: I will discuss my result with B. Mance on the Hausdorff dimension of the set of numbers with bounded Lüroth expansions. In particular, this set is dense and has full Hausdorff dimension and a countable intersection property. In fact, this set is winning, which is a property associated with the notion of Schmidt games. The proof is an adaptation of my technique for C^2-expanding circle maps and the main obstacle in using this technique is the handling of an infinite partition that encodes the expansion.

The bounded Lüroth expansions result is part of a larger theme in my research of studying the fractals that encode nondense orbits of interesting ergodic dynamical systems. I have a number of results in this theme such as my results with V. Bergelson and M. Einsiedler and my result with R. Shi. I will, however, focus on two of my recent solo results, one on nondense orbits for C^2-Anosov diffeomorphisms of the 2-torus and one on simultaneous dense and nondense orbits for pairs of C^2-Anosov diffeomorphisms of the 2-torus, because the techniques used in these latter two results are more closely related to the bounded Lüroth expansions result.

Sandro Vaienti (Université de Toulon): Wed 11:00 – 12:00

Title: On a few statistical properties of sequential and random fibred systems

Abstract: We present some new results for statistical properties of non-autonomous dynamical systems, including sequential systems and random fibred systems. We will investigate in particular the almost sure invariance principle and the Extreme Value Distribution.

Reem Yassawi (Trent University): Thu 11:00 – 12:00

Title: Dynamical systems on $\mathbb{Z}_p$ generated by constant length substitutions

Abstract: Motivated by natural combinatorial sequences of integers, we define constant length substitutions on a closed subset of the p-adic integers $\mathbb{Z}_p$, for p prime. We show that these substitutions have fixed points generated by *profinite* automata, and generate shifts that often share several dynamical properties of constant length substitution shifts on a finite alphabet. This is joint work with Eric Rowland.

Yuru Zou (Shenzhen University): Fri 11:00 – 11:45

Title: On a problem of countable expansions

Abstract: For a real number $q \in (1,2)$ and $x \in [0, 1/(q-1)]$, the infinite sequence (d_i) is called a *q-expansion* of x if

$$x = \sum_{i=1}^{\infty} \frac{d_i}{q^i}, \quad d_i \in \{0,1\} \quad \text{for all } i \geq 1.$$

For $m = 1, 2, \cdots$ or $\aleph_0$ we denote by B_m the set of $q \in (1,2)$ such that there exists $x \in [0, 1/(q-1)]$ having exactly m different q-expansions. It was shown by Sidorov [Nikita Sidorov, Expansions in non-integer bases: lower, middle and top orders, J. Number Theory 129(2009), 741-754] that $q_2 := \min B_2 \approx 1.71064$, and later asked by Baker [Simon Baker, On small bases which admit countably many expansions, J.Number Theory 147(2015), 515-532] whether $q_2 \in B_{\aleph_0}$? In this talk we will provide a negative answer to this question and conclude that $B_{\aleph_0}$ is not a closed set. In particular, we will give a complete description of $x \in [0, 1/(q_2 - 1)]$ having exactly two different q_2-expansions.