Learning Seminar on Elliptic Units

Goal: Define elliptic units and prove their norm relations. This gives rise to the Euler system of elliptic units, which is the key ingredient in the approach of Coates and Wiles to proving the Birch and Swinnerton-Dyer conjecture for elliptic curves with complex multiplication in analytic rank zero. Depending on time and interest, we could try to understand the connection with L-functions and prove the Coates–Wiles theorem.

When and where: Sundays 10-11am in Room 209 of Manchester Building (HUJI). Starting date: Sunday May 8.

Main reference: Sections 5-10 of Rubin's Cetraro lecture notes https://www.math.arizona.edu/~swc/aws/1999/99RubinCM.pdf.

Other references:

- The original work of Coates and Wiles
  On the conjecture of Birch and Swinnerton-Dyer, Invent. Math. **39**, (1977), 223–251.

- Chapter 1 of the expository article by Bertolini et. al. on Euler systems
  https://www.math.mcgill.ca/darmon/pub/Articles/Research/61.Durham-ES/durham.pdf
  (mostly for context and broader connections with Euler systems).

- Sections 1-4 of Rubin's notes
  https://www.math.arizona.edu/~swc/aws/1999/99RubinCM.pdf
  (background material).

Talks:

- **May 8:** Introduction   (David Lilienfeldt)
  Motivation: elliptic curves, BSD conjecture, Euler systems. Outline of program.

- **May 15:** Elliptic curves with complex multiplication
  Basic definitions and properties following §5.1 (pg. 15–17). For the necessary background, draw from §1–4.

- **May 22:** Main theorem of CM theory
  Formulate the main theorem in the language of Shimura and define the Hecke character associated with a CM elliptic curve following §5.2 (pg. 17–22).

- **May 29:** Basic properties of elliptic units
  Define elliptic units and show that they are global units in ray class fields of imaginary quadratic fields, following §7.1. Prove the norm relations following §7.2 & 7.3.

- **June 5:** Elliptic units and L-functions
  Following §7.4, prove Theorem 7.17 relating elliptic units to special values of Hecke L-functions. Deduce the p-adic analogue following §7.5.

- **June 12:** The Euler system of elliptic units
  Following §8.

- **June 19:** Bounding ideal class groups using Kolyvagin's method
  Following §9.

- **June 26:** The theorem of Coates–Wiles
  Following §10. Introduce the necessary background on Selmer groups following §6.

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