Largeness and deficiency in infinite groups

Rachel Newton

A presentation P for a group G is a set of generators and relations for G. Presentations can be deceptive. Consider the following two presentations:

$$P_1 = \langle a, b, c | b^{-1}ab = a^2, c^{-1}bc = b^2, a^{-1}ca = c^2 \rangle$$

$$P_2 = \langle a, b, c, d | b^{-1}ab = a^2, c^{-1}bc = b^2, d^{-1}cd = c^2, a^{-1}da = d^2 \rangle$$

They may look similar, but P_1 gives the trivial group while P_2 gives an infinite group.

Nonetheless, there are invariants associated to a presentation which give valuable information about the group. Deficiency is one such invariant. The deficiency of a presentation is defined to be the number of generators minus the number of relators. It is a direct consequence of Smith normal form that positive deficiency implies that the group is infinite. But deficiency can tell us much more about what kind of infinite group it is. For example, whether it has a property called 'largeness'.

In this project, you will learn what largeness is and explore the link between largeness and deficiency.