Rational points on varieties, part II (surfaces)

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1. Brauer–Manin obstruction to the Hasse principle

- Example of failure of the Hasse principle on a del Pezzo surface of degree 4, [4, Wed, 3rd].
- Example of failure of weak approximation on a singular cubic surface, [4, Wed, 3rd].
- Hilbert symbols, quaternion algebras, central simple algebras, Brauer group, [4, Thu, 1st].
- Brauer–Manin obstruction to the Hasse principle, [4, Thu, 2nd].

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