Linear algebra 2: exercises for Section 5

Ex. 5.1. In each of the following cases indicate whether there exists a real 4×4-matrix A with the given properties. Here I denotes the 4×4 identity matrix.

1. $A^2 = 0$ and A has rank 1;

2. $A^2 = 0$ and A has rank 2;

3. $A^2 = 0$ and A has rank 3;

4. A has rank 2, and $A - I$ has rank 1;

5. A has rank 2, and $A - I$ has rank 2;

6. A has rank 2, and $A - I$ has rank 3.

(HW) Ex. 5.2. For the following matrices A, B give their Jordan normal forms, and decide if they are similar.

$$A = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & 1 & 2 & -1 \\ 0 & 0 & 2 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 0 & 0 & -2 \\ 1 & 2 & 1 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

Ex. 5.3. Give the Jordan normal form of the matrix

$$\begin{pmatrix} 2 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 1 & 5 & 2 & -2 \\ 0 & -4 & 0 & 4 \end{pmatrix}$$

Ex. 5.4. Give the Jordan normal form of the matrix

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

(HW) Ex. 5.5. Let A be the 3×3 matrix

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}.$$

Compute A^{100}.

Ex. 5.6. Consider the matrix $A = \begin{pmatrix} 1 & 4 \\ -1 & 5 \end{pmatrix}$.

1. Give the eigenvalues and eigenspaces of A.

2. Give a diagonal matrix D and a nilpotent matrix N for which $D + N = A$ and $DN = ND$.

3. Give a formula for A^n when $n = 1, 2, 3, \ldots$

(HW) Ex. 5.7. For the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

give a diagonalizable matix D and a nilpotent matrix N so that $A = D + N$ and $ND = DN$.

Ex. 5.8. For $A = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 4 & -2 \\ 0 & 2 & 0 \end{pmatrix}$ compute the matrix e^A.