

## Linear algebra 2: exercises for Chapter 4

**Ex. 4.1.** Let  $A$  be a nilpotent  $n \times n$  matrix. Show that  $\text{id}_n + A$  is invertible.

**Ex. 4.2.** Let  $A$  be a nilpotent  $n \times n$  matrix. Show that  $A^n = 0$ .

**Ex. 4.3.** Let  $N$  be a  $9 \times 9$  matrix for which  $N^3 = 0$ . Suppose that  $N^2$  has rank 3. Prove that  $N$  has rank 6.

**Ex. 4.4.** Let  $N$  be a  $12 \times 12$  matrix for which  $N^4 = 0$ .

1. Show that the kernel of  $N^2$  contains the image of  $N^2$ .
2. Show that the rank of  $N$  is at most 9.
3. Show that the rank of  $N$  is equal to 9 if the kernel of  $N^2$  is equal to the image of  $N^2$ .

**Ex. 4.5.** For which  $x \in R$  is the following matrix nilpotent?

$$\begin{pmatrix} 2x & x & -1 \\ -4 & -1 & -3 \\ 5 & 2 & 3 \end{pmatrix}$$

**Ex. 4.6.** For each of the matrices

$$\begin{pmatrix} 4 & -4 & 12 \\ 1 & -1 & 3 \\ -1 & 1 & -3 \end{pmatrix} \quad \begin{pmatrix} 2 & 0 & 8 \\ 0 & 1 & 1 \\ -1 & 1 & -3 \end{pmatrix}$$

give a basis of  $\mathbb{R}^3$  for which the matrix sends each basis vector either to 0 or to the next basis vector in the basis.

**Ex. 4.7.** Do the same for the matrix

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ -5 & -2 & 2 & -1 \\ -3 & 0 & 2 & -1 \\ -5 & -2 & 2 & -1 \end{pmatrix}$$