EXTRA OPGAVEN BILINEAIRE VORMEN

RONALD VAN LUIJK

(1) Let $\phi\colon \mathbb{R}^4 \times \mathbb{R}^3 \to \mathbb{R}$ be the bilinear form given by $(x, y) \mapsto y^\top Ax$ with

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{pmatrix}.$$

Let $f\colon \mathbb{R}^4 \to \mathbb{R}^4$ be the isomorphism given by

$$(x_1, x_2, x_3, x_4) \to (x_1, x_1 + x_2, x_1 + x_2 + x_3, x_1 + x_2 + x_3 + x_4).$$

Let $g\colon \mathbb{R}^3 \to \mathbb{R}^3$ be the isomorphism given by

$$(x_1, x_2, x_3) \to (x_1, x_1 + x_2, x_1 + x_2 + x_3).$$

Let $b\colon \mathbb{R}^4 \times \mathbb{R}^3 \to \mathbb{R}$ be the map given by $b(x, y) = \phi(f(x), g(y))$.

(a) Determine the kernel of ϕ_L and ϕ_R.
(b) Show that b is bilinear.
(c) Give the matrix associated to b with respect to the standard bases for $\mathbb{R}^4$ and $\mathbb{R}^3$.

(2) Let V be a finite-dimensional vector space over F, and $\mathrm{ev}\colon V \times V^* \to F$ the bilinear form that sends (v, φ) to $\varphi(v)$. Let B be a basis for V, and B^* its dual basis for V^*. What is the matrix associated to ev with respect to the bases B and B^*?

(3) Verify Example 8.16.