Extra opgaven hoofdstuk 6, Lineaire Algebra 2

(1) Suppose we have a long exact sequence

$$0 \longrightarrow V_1 \longrightarrow V_2 \longrightarrow \cdots \longrightarrow V_n \longrightarrow 0$$

of vector spaces. Show that we have $\sum_{i=1}^{n} (-1)^i \dim V_i = 0$.
[Hint: first do the case $n = 3$].

(2) Suppose $f\colon U \to V$ and $g\colon V \to W$ are linear maps such that

$$U \xrightarrow{f} V \xrightarrow{g} W \longrightarrow 0$$

is an exact sequence. Suppose that $F_U\colon U \to U$ and $F_V\colon V \to V$ are endomorphisms such that $F_V \circ f = f \circ F_U$. Show that there exists an endomorphism $F_W\colon W \to W$ such that $F_W \circ g = g \circ F_V$. In other words, show that there exists an endomorphism F_W of W such that the following diagram commutes.

$$\begin{array}{ccccccc} U & \xrightarrow{f} & V & \xrightarrow{g} & W & \longrightarrow & 0 \\ \downarrow{\scriptstyle F_U} & & \downarrow{\scriptstyle F_V} & & \downarrow{\scriptstyle F_W} & & \\ U & \xrightarrow{f} & V & \xrightarrow{g} & W & \longrightarrow & 0 \end{array}$$