GLOBAL ANALYSIS I - WS 2017/2018

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Exercise sheet 8 - due Wednesday 13-12 -2017

Please hand in the exercises stapled or paperclipped together, with name AND Matrikelnr. If you work in groups, one sheet per group suffices (at most 3 people per group).

1. Exercise (Cf. Exercise 4.5 of *Riemannian manifolds*)**.** Let ∇ be a linear connection on M, E_i a local frame and ϕ_i its dual coframe.

(1) Show that there exists a uniquely determined matrix of 1-forms ω_{ij} such that for all $X \in \mathscr{X}(M)$

$$\nabla_X(E_i) = \sum_j \omega_{ij} E_j.$$

The ω_{ij} are called the *connection 1-forms* for the frame E_i.

(2) Let τ be the torsion tensor and τ_j the *torsion 2-forms* defined by

$$\tau(X,Y) = \sum_j \tau_j(X,Y) E_j.$$

Prove *Cartan's first structure equation*:

$$d\phi_j = \tau_j + \sum_i \phi_i \wedge \omega_{ij}.$$

2. Exercise (Cf. Exercise 5.1 of *Riemannian manifolds*)**.** Let ∇ be a linear connection on a Riemannian manifold (M,g). Show that ∇ is compatible with g if and only if the connection 1-forms ω_{ij} with respect to a local frame E_i satisfy

$$\sum_k g_{jk}\omega_{ki} + g_{ik}\omega_{kj} = dg_{ij},$$

where g_{ij} are the components of g with respect to the frame E_i. In particular the matrix ω_{ij} with respect to any local frame is skew symmetric.