Assignment-set 1 Bifurcations and Chaos

Deadline to hand in: 22 March 2018, 11.00u

1.) Consider the following equation

$$\frac{dx}{dt} = -\varepsilon x \cos^2(t)$$

where $0 < \varepsilon \ll 1$ and $x = x_0$ at $t = 0$.

(a) Find the exact solution of this equation.

(b) Determine the averaged equation and solve it.

(c) Compare the results of (a) and (b). How large is the error incurred by averaging? Does this correspond to the averaging theorem? Sketch both solutions in one plot.

2.) Consider

$$\frac{dx}{dt} = \varepsilon(a + \sin t - x)$$

where $0 < \varepsilon \ll 1$ and $a \in \mathbf{R}$.

(a) Show that there exists a periodic solution and give an expression for the solution.

(b) Is this solution stable or unstable?

3.) Consider

$$\begin{aligned} \frac{dx}{dt} &= y + \varepsilon(x^2 \sin(2t) - \sin(2t)) \\ \frac{dy}{dt} &= -4x \end{aligned}$$

where $0 < \varepsilon \ll 1$.

(a) Determine the averaged system of equations (in suitable variables).

(b) Do there exist periodic solutions to the original system? If so, determine their stability.

4.) Guckenheimer and Holmes exercise 4.5.1 on p190.