Assignment-set 4 Introduction to Dynamical Systems 2014

Deadline to hand in: 17 December 2014, 9.00u, in mailbox Corine

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1.) Consider the planar problem

$$\begin{cases} \dot{x} &= -\frac{\partial E}{\partial y} + \lambda E \frac{\partial E}{\partial x}, \\ \dot{y} &= \frac{\partial E}{\partial x} + \lambda E \frac{\partial E}{\partial y}, \end{cases} \tag{1}$$

with $E(x,y) = -2x^2 + x^4 + y^2$ and $\lambda \in \mathbf{R}$.

(a) Determine the critical points and their local character for all $\lambda \in \mathbf{R}$. What is special about the case $\lambda = 0$?

(b) Let $\gamma_i^+(t)$ $(t \geq 0)$ be the solution of (1) with $\gamma_i^+(0) = (x_i, y_i)$, $i = 1,2,3$, and $(x_1, y_1) = (\frac{1}{2}, 0)$, $(x_2, y_2) = (-\frac{1}{2}, 0)$, $(x_3, y_3) = (0,1)$. Determine $\omega(\gamma_i^+) = \omega((x_i, y_i))$ for $i = 1,2,3$ and for all $\lambda \in \mathbf{R}$.
Hint: Determine and use $\dot{E}$.

2.) Consider the system

$$\begin{cases} \dot{x} &= \cos(2\pi t)x, \\ \dot{y} &= \sin(2\pi t)x + (\cos(2\pi t) - 1)y. \end{cases}$$

(a) Find the Floquet multipliers of the system.

(b) Determine the stability of the origin.

3.) Consider the 2-dimensional system

$$\begin{cases} \dot{x} &= \beta - x + x^2 + xy, \\ \dot{y} &= 2y + x^2 - y^2. \end{cases} \tag{2}$$

(a) Take $\beta = 0$. Show that (2) has two saddle points $P^+(0)$ and $P^-(0)$ and that these points are connected by a heteroclinic orbit. Give a sketch of the phase portrait.

(b) Now consider $\beta \neq 0$ and small. Determine a Taylor/perturbation expansion of the saddles $P^+(\beta)$ and $P^-(\beta)$ up to and including quadratic terms in β. What happens to the heteroclinic connection? Give a sketch of the phase portraits, both for $\beta > 0$ as well as for $\beta < 0$.

4.) Consider the equation

$$y'' + y' + y[2y^2 + \lambda(\lambda - 2)] = 0,$$

for $\lambda \in \mathbf{R}$

(a) Determine the steady states and their stability.

(b) Sketch the steady states in the (λ, y)-plane. Also denote the stability of the steady states in this sketch by dashing the unstable solutions and plotting the stable solutions by a solid line. This sketch is a so-called bifurcation diagram which displays how solutions change as the parameter λ is varied.

(c) Take $\lambda > 1$. Sketch the phase plane for the ranges of λ where qualitatively different dynamics is observed.