Assignment-set 1 Introduction to Perturbation Methods

Deadline to hand in: 9 March 2017, 11.15u

1.) Consider the following equation

$$\varepsilon x^3 - 4x + \varepsilon = 0$$

where $0 < \varepsilon \ll 1$. Determine a two-term approximation of all the roots of the equation.

2.) Consider the following equation

$$x^2 + \varepsilon^2\sqrt{2+x} = \cos(\varepsilon)$$

where $0 < \varepsilon \ll 1$.

(a) Determine the number of solutions of this equation.

(b) Determine a two-term approximation of all the roots of the equation.

3.) Consider the initial value problem

$$\frac{d^2y}{dx^2} + y + \varepsilon y^3 = 0$$

with $y(0) = 0$ and $y(\frac{\pi}{2}) = 1$. Determine a two-term approximation of the solution to this problem.

4.) Holmes exercise 1.37 on p42-43.

5.) Find a composite expansion of the following boundary value problem

$$\varepsilon\frac{d^2y}{dx^2} + (1+2x)\frac{dy}{dx} - 2y = 0$$

for $0 < x < 1$ with $y(0) = \varepsilon$ and $y(1) = \sin(\varepsilon)$.