Assignment-set 5 Applied Analysis

Due on 17 December 2007, 10.00u

1.) Consider the following equation

$$\varepsilon^2 x^3 - x + \varepsilon = 0$$

where $0 < \varepsilon \ll 1$. Determine a two-term approximation of all the roots of the equation.

2.) Consider the initial value problem

$$\varepsilon \frac{d^2 y}{dx^2} - 2\frac{dy}{dx} - y^3 = 0$$

with $y(0) = \frac{1}{3}\sqrt{3}$ and $y(1) = 0$.

(a) Substitute the regular expansion $y(x) = y_0(x) + \varepsilon y_1(x) + \ldots$ into the equation and determine y_0 and y_1. Conclude that there exists a boundary layer at $x = 1$.

(b) Introduce $z = \frac{x-1}{\varepsilon^\mu}$ and $Y(z) = y(x)$. Choose μ in a suitable way and determine the first two terms in the expansion of $Y(z)$.

(c) Give a formal expansion of the initial value problem by matching the solutions found in a. and b.