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Studiegroep wiskunde in de industrie

A greenhouse growth model for unselfish roses

In februari 2002 vond de tweeënveertigste Studiegroep Wiskunde met de Industrie plaats. Een van de problemen betrof het modelleren van de groei van kasrozen. Het uiteindelijke doel van de studie is het construeren van een model dat door middel van terugkoppeling het klimaat in een kas zo regelt dat een optimale groei wordt verkregen. Het probleem is aangedragen door het bedrijf Phytocare, een adviesbureau voor rozenkwekers.

In recent years, production of roses has become increasingly more competitive and commercialised. Whilst the rose-grower's experience remains the key to producing a large rose harvest, qualitative and quantitative modelling of the biochemical processes continues to take an ever more prominent role in optimising production even further. Nowadays, it is commonplace for the internal climate of a greenhouse to be computer-controlled, although the question of how to develop a climate control model to maximise production cannot yet be answered. Indeed, maintaining a constant internal climate is not enough, as the model must react to changes in weather and seasonal climate outside the greenhouse. Sudden rainfall, for instance, can lead to a sharp drop in temperature, which can, in turn, adversely affect the rose harvest some days or weeks later.

Our task during the study group was to develop a mathematical model of rose production, which given the internal climate conditions can predict the total mass of rose harvest produced per square metre of greenhouse per week. It is hoped that eventually the model will enable the rose grower to tune these climatic conditions inside the greenhouse dynamically to maximise the rose harvest.

Leaf photosynthesis

Rose stems grow by assimilating CO_2 from the air. This process is performed in the leaves and is called photosynthesis. The CO_2 assimilation and, therefore, the growth of the roses is influenced by several environmental factors. Some of these can be controlled by the rose-grower, for example by using heaters or lamps, opening or closing the windows and putting up blinds for shade. These actions, in turn, alter the CO_2 concentration in the air C_a (by ventilation), the relative humidity R_H, the temperature in the greenhouse T_a, and the overall light intensity I_0.

We began the project with a model, derived from [2–3], that calculates the photosynthetic rate of a single leaf given its age a, the greenhouse temperature, humidity and CO_2 concentration, as well as the amount of light intensity I the leaf receives. This leaf model was very intricate and our initial task involved unravelling all the effects of climate and leaf properties on the local photosynthetic rate. In fact, the photosynthetic rate P per unit area of leaf can be written as an equation of the form

$$P = f(P, a, T_a, R_H, C_a, I),$$

where f is explicitly known and is a non-linear function. It was found that this model could be simplified a great deal and linearised in various limits, whilst retaining the most important biological properties.

The greenhouse model

Given the photosynthesis model for a single leaf, our main challenge for the study group was to use this leaf model to determine how an entire rose crop inside a greenhouse would grow under certain climatic conditions. It was clear from the photosynthesis model that the task would not be completely straightforward, as the amount of biomass produced depends on the age of the leaf and on the amount of light each leaf receives. Both of these variables will clearly change dynamically in time as the rose plants grow, producing new young leaves whilst increasingly shading the lower leaves from the light above.

Model assumptions

To model the rose plants in the greenhouse, we assume that each plant can be divided into two constituent parts: (i) the *rose crop* above which consists of one or more rose stems to be harvested and (ii) the *rose bush* below that supports the stems but is not harvested. A rough sketch of our idealised greenhouse can be seen in figure 1.

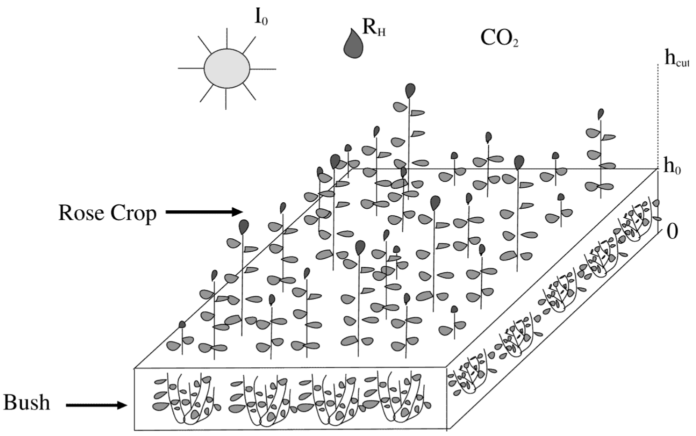

Figure 1 A sketch of an idealised greenhouse, where R_H denotes the relative humidity, I_o the overall light intensity, h_o the level at which the harvested stems are cut on reaching height h_{cut}.

The bush is taken to have a constant height h_0 and contains leaves that assimilate and thus contribute to the total photosynthetic rate of the greenhouse. The rose stems grow vertically out of the rose bush and are harvested once they reach a certain height h_{cut}. Furthermore, the harvested stems are all cut at the level h_0, so that they are all of the same length, $h_{cut} - h_0$. As the mature roses are cut, new stems are assumed to grow out of the top of the bush appearing at a rate proportional to the current net photosynthetic rate in the greenhouse. For simplicity, we ignore the part of the acquired photosynthetic energy that is used for maintenance, storage and blossoming, and assume that the photosynthesis in the crop and bush is entirely used to increase the mass of stems in the crop.

As the photosynthesis in a leaf depends on the age of the leaf, we have to know where the young and old leaves are positioned on a rose plant. For this reason we assume that any new leaves appear at the top of the rose stems, agreeing reasonably well with observations. We also suppose that the leaves, and therefore the leaf area, are distributed uniformly along the stem. In other words, the leaf area of each stem is proportional the stem's length. As the rose-grower is concerned with the productive mass of the crop, we also assume that the mass of each rose stem is proportional to its length.

The principle of unselfishness

A further assumption which simplifies the greenhouse model greatly is the so-called *unselfishness principle*. The principle states that any biomass obtained from photosynthesis of a single leaf, either located on a stem or within the bush, contributes equally to the growth of all rose stems, whether large or small. Hence, although a taller stem, which has more leaves, will assimilate more CO_2 than a shorter stem, the combined biomass produced will be shared equally between them. As a result, every stem grows at the same speed, independent of its own photosynthetic rate.

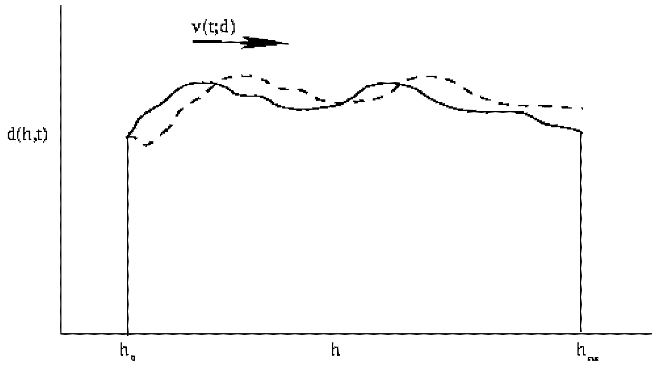

Figure 2 Every rose grows at the same rate. Here $d(h,t)$ denotes the number of stems per square metre of greenhouse as a function of height h and time t and $v(t;d)$ denotes the growth rate by which the density function is advected.

The unselfishness principle reflects both real data and the observation that a single rose plant, possessing a number of rose stems of differing heights, acts as a single entity. In this way new young stems can develop quickly, even though they do not possess a large leaf area.

The growth equation

From these assumptions, the state of the rose crop at any given instant of time can be described by a stem density function $d(h,t)$, representing the distribution of stems of differing heights. The function $d(h,t)$ describes the number of stems per square metre of greenhouse as a function of height h and time t. From the unselfishness principle stating that every rose stem grows at the same speed, it follows that this density function is advected by a growth rate $v = v(t;d)$, which is *independent* of h (see figure 2). The dynamics of d can be written down in the following way

$$\frac{\partial d}{\partial t} + v\,\frac{\partial d}{\partial h} = 0.$$

The equation requires an initial state of the greenhouse rose crop at a given time and a boundary condition at $h = h_0$, representing the creation of new stems from the rose bush. From an assumption already stated, it follows that these new stems are created at a rate proportional to the net photosynthesis denoted by $P_{net}(t)$. This photosynthetic rate represents the net biochemical intake or loss of CO_2 per square metre of greenhouse from both bush and crop.

Since roses are cut once they have reached height h_{cut}, the harvest rate $H(t)$ per square metre of greenhouse is given by

$$H(t) \propto v(t;d)(h_{cut} - h_0)\, d(h_{cut}, t),$$

where, from here on, $\propto$ means 'proportional to'.
Note that this harvest rate is exactly what the rose-grower wishes to maximise.

Determining the growth speed

The growth speed v is determined by using a mass balance. The net photosynthetic rate P_{net} changes not only due to the greenhouse climate, but also because of growth of the roses (crop and bush) and the harvesting of the rose crop. More specifically, P_{net} is proportional to the change in productive mass plus the harvest rate

$$P_{net}(t;d) \propto \frac{dM}{dt} + H(t). \tag{1}$$

Here M is the total mass of the rose crop and it is given by

$$M(t) \propto \int_{h_0}^{h_{cut}} (h - h_0)\, d(h,t)\, \mathrm{d}h,$$

where the stem density is properly weighed by the stem length $h - h_0$ in order to obtain the mass.

Differentiating this expression for M leads, after integrating by parts, to

$$\frac{dM}{dt} \propto v(t;d) \int_{h_0}^{h_{cut}} d(h,t)\,\mathrm{d}h - H(t).$$

By substituting this into (1) and solving for v, we obtain

$$v(t;d) \propto \frac{1}{\int_{h_0}^{h_{cut}} d(h,t)\,\mathrm{d}h} P_{net}(t;d), \tag{2}$$

as the advection speed in terms of P_{net}.

The net photosynthetic rate

Our next step is to obtain an expression for the net photosynthetic rate P_{net} in (2). This must be determined by adding the local photosynthetic contribution from each leaf in the rose crop and the rose bush. Thus, we have

$$P_{net} = P_{crop} + P_{bush},$$

where the contributions from the crop and the bush are to be calculated separately.

As the leaf's local photosynthetic rate depends both on its age and on the amount of light it receives, an ability to model the age and height distribution of leaves is very important. To begin with, we define a leaf-density function $\rho(h,t)$, where $\rho(h,t)\,\mathrm{d}h$ yields the leaf area located at heights between h and $h+\mathrm{d}h$ per square metre of greenhouse. For $h_0 < h < h_{cut}$, it is related to $d(h,t)$ by

$$\rho(h,t) = k_\rho \int_{h}^{h_{cut}} d(\zeta,t)\,\mathrm{d}\zeta,$$

(k_ρ is a constant of proportionality) using the fact that only rose stems of heights greater than h contribute to the leaf area at height h.

Furthermore, we define the age density distribution $q(t,a,h)$ in such a way that $q(t,a,h)\,\mathrm{d}h\,\mathrm{d}a$ yields the total leaf area of ages between a and $a+\mathrm{d}a$ located at heights between h and $h+\mathrm{d}h$ per square metre of greenhouse. Physically, it is clear from our model assumptions that the age of a leaf bears some relation to its distance from the top of the rose stem (see figure 3). By simplifying this relationship to assume that the age of a leaf is directly proportional to this distance, we can deduce that

$$q(t,a,h) \propto d\left(h + \frac{a}{k}, t\right)$$

for an average inverse growth speed k.

foto: Nick Ovenden

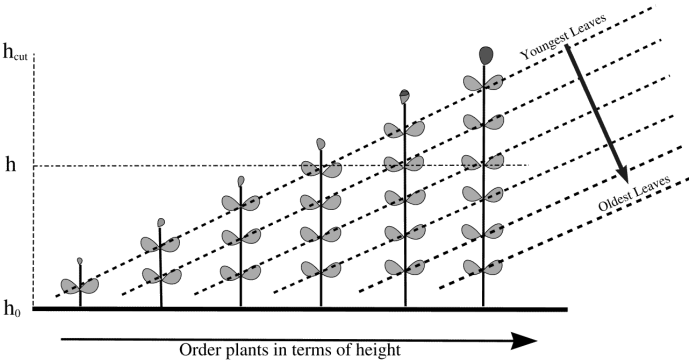

Figure 3 The age of a leaf is assumed to be proportional to its distance from the top of the rose stem.

The local light intensity reaching a particular leaf depends on the amount of shading or, in other words, on the leaf coverage (leaf area) above it. Observing that all stem heights tend to be distributed evenly throughout the greenhouse suggests that all leaves at the same height have approximately the same amount of shade. Therefore, the change in light intensity $\frac{\mathrm{d}I}{\mathrm{d}h}$ between h_0 and h_{cut} is a function of h as well as t and taken to be proportional to $\rho(h,t)$ and $I(h,t)$ (with proportionality constant k_I), leading to

$$I(h,t) = I_0(t)\, e^{-k_I \int_h^{h_{cut}} \rho(\zeta,t)\,\mathrm{d}\zeta}.$$

Now, the net photosynthesis produced per square metre in the crop follows by integration of the local rate of photosynthesis over all the leaf ages and heights in the crop, weighted by the age density distribution:

$$P_{crop}(t;d) = \int_{h_0}^{h_{cut}} \int_0^{T_{max}} q(t,a,h)\, P(t,a,h)\, \mathrm{d}a\, \mathrm{d}h.$$

The time T_{max} is the age of the oldest leaf in the current rose crop and is comparable to the average total growth time of a stem from its first appearance on the bush to harvest; T_{max} differs for each type of rose and each growth season. The net photosynthetic rate in the bush P_{bush} still needs to be determined to obtain P_{net} and this requires some knowledge of the leaf distribution in the unharvested part of each rose plant. Indeed, what is needed are expressions for $\rho(h,t)$ and $q(t,a,h)$ in the bush and the actual position of the bush in relation to the crop (directly below the stems or overhanging on the side). One simple bush model adopted during the study group was to assume that leaves of different ages were uniformly distributed throughout the bush between $h = 0$ and $h = h_0$. Moreover, for the purposes of gauging the amount of light received, we assumed the bush to be located directly below the crop (well shaded). Other bush models are equally possible to implement into our model and, for the rose-grower, the question of how the bush part should be grown and maintained is fundamental to the aim of continuously maximising the rose harvest.

Fitting the data

So far, we have obtained a rose production model for a greenhouse. In this model, several proportionality constants still appear as parameters. Most of these parameters can be determined by the rose-grower either by direct measurement of the rose plants or by relating the theoretical harvest and rates of new stem appearance to (seasonally or yearly) averaged harvest data. The remaining parameters can be obtained by fitting the model to weekly harvest data. In [1], we describe how the estimation of these parameters can be accomplished. The model should then, in principle, be able to aid the rose-grower to optimise the weekly amount of harvested roses. However, a comparison between data and the model, as well as the desired optimisation of the rose production by running the model in a forecasting mode, is still work in progress.

Future work

Some of our modelling assumptions are an oversimplification of the real situation. One drawback in our approach seems to be the fact that the only measure for the development of a rose stem is the total conversion of photosynthetic energy into stem length. This is not very realistic, as can be seen for example from the seasonal differences in the thickness of the harvested rose stems. In particular, the process of blossoming, which is a crucial guide as to when the rose should be cut, is not modelled at all. Neither is the diverting of photosynthetic energy into storage and maintenance of the rose plant. To model the roses more realistically, some detailed knowledge of how the total photosynthesis is allocated to growing, producing more leaves, generating new stems (which is only basically modelled here) and blossoming is required. Models of these processes in single rose plants are beginning to appear in the literature and should be investigated further.

Despite these shortcomings, we hope that our modelling approach via stem, leaf, and age density functions will prove flexible enough to be coupled to more complex and precise rose growth models, leading eventually to an accurate interactive model of an entire greenhouse. One benefit of our greenhouse model to the industrial partner (*Phytocare*) is that it can be compared to the real greenhouse data that is measured; this is something that could not be done with a single leaf model. Further improvements to the model, together with the continuation of the work on parameter estimation and bush design, form an intriguing challenge for further research in optimising rose production.

Acknowledgements

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