Homework set 4 due December 5 2011

Exercise 4.1. For an abelian group A with endomorphisms $f, g\colon A \to A$ satisfying $fg = gf = 0$ we define

$$q_{f,g}(A) = [\mathrm{Ker} f : \mathrm{Im} g]/[\mathrm{Ker} g : \mathrm{Im} f]$$

whenever both indices are finite.

1. For $n \in \mathbb{Z}$ compute $q_{0,n}(\mathbb{Z})$.
2. Show that $q_{f,g}(A)$=1 whenever A is finite.
3. Show how the usual Herbrand quotient $h(M)$ of a module M over a finite cyclic group G is a special case of this.

Exercise 4.2. Let K be an algebraically closed field, let m be a non-negative integer and let μ_m denote the group of mth roots of unity in K. Let G be a group of automorphisms of K of order m. Show that the map $H^2(G, \mu_m) \to H^2(G, K^*)$ is an isomorphism.

Exercise 4.3. Let k be a finite field and let $\overline{k}$ be its algebraic closure. Show that $H^2(\mathrm{Gal}(\overline{k}/k), \overline{k}^*) = 0$.

Exercise 4.4. Let G be a profinite group and M a finitely generated discrete G-module which is torsion free as an abelian group. Does this imply that $H^1(G, M)$ is finite?