Homework set 5 due December 19 2011

Exercise 5.1. Let p be a prime number below 50 and suppose that K is an abelian extension of $\mathbb{Q}_p$ of degree 15 which is totally ramified. Show that $p = 31$.

Exercise 5.2. Give an example of a prime p, a finite extension K of $\mathbb{Q}_p$ and an abelian extension L/K of exponent 2 and degree 64.

Exercise 5.3. Let K be a local field of characteristic 0. Show that every finite index subgroup of K^* is open. Bonus question: is the same true for characteristic p?

Exercise 5.4. Let K be a local field, let K^{sep} be the separable closure of K, and let G_K be the profinite group $\text{Gal}(K^{\text{sep}}/K)$. Show that $H^3(G_K, K^{\text{sep}*}) = 0$.

Exercise 5.5. Let p be a prime, $K = \mathbb{Q}_p$, and $n = p-1$. Recall that $\mu_n \subset K^*$. For $a, b \in K^*$ let $(a, b) \in \mu_n$ be the image of the pair $(a \bmod (K^*)^n, b \bmod (K^*)^n)$ under the Hilbert symbol $K^*/(K^*)^n \times K^*/(K^*)^n \to \mu_n$. Show that $(a, b) \equiv (-1)^{v(a)v(b)} a^{v(b)} b^{-v(a)} \bmod p\mathbb{Z}_p$. Hint: use Example 3.13 of Chapter 2, and you may also use that $\mathbb{Q}_p(\zeta_p) = \mathbb{Q}_p(\sqrt[p-1]{-p})$.