Linear algebra 2: exercises for Section 10

Ex. 10.1. Suppose that A is a symmetric 2×2 matrix of determinant 2 for which $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ is an eigenvector with eigenvalue -1.

1. What is the other eigenvalue of A?
2. What is the other eigenspace?
3. Determine A.

Ex. 10.2. Consider the quadratic form $q(x, y) = 11x^2 - 16xy - y^2$.

1. Find a symmetric matrix A for which

$$q(x, y) = (x\ y) \cdot A \cdot \begin{pmatrix} x \\ y \end{pmatrix}.$$

2. Find real numbers a, b and an orthogonal map $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ so that $q(f(u, v)) = au^2 + bv^2$ for all $u, v \in \mathbb{R}$.
3. What values does $q(x, y)$ assume on the unit circle $x^2 + y^2 = 1$?

Ex. 10.3. What values does the quadratic form $q(x, y, z) = 2xy + 2xz + y^2 - 2yz + z^2$ assume when (x, y, z) ranges over the unit sphere $x^2 + y^2 + z^2 = 1$ in $\mathbb{R}^3$?

Ex. 10.4. Suppose that A is an anti-symmetric $n \times n$ matrix over the real numbers.

1. Show that every eigenvalue of A over the complex numbers lies in $i\mathbb{R}$.
2. If n is odd, show that 0 is an eigenvalue of A.