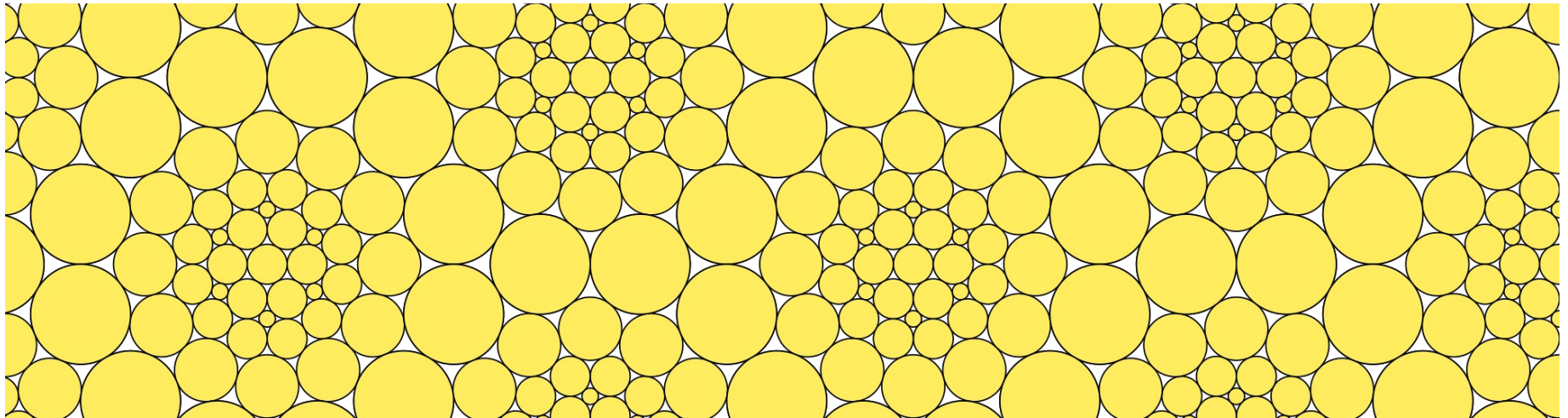


Compact packings of the plane with discs

Miek Messerschmidt

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Acknowledgment: Marcel de Jeu, Erasmus+ ICM



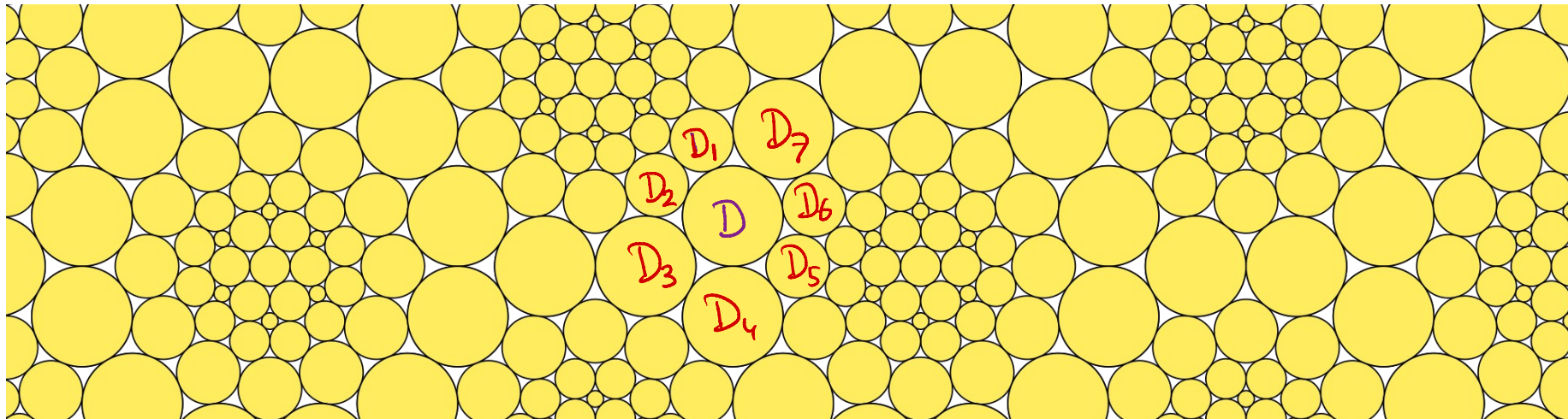
Bachelorproject.

Begeleider : Marco Streng

Def Een cirkelstapeling is een verzameling S van gesloten schijven (discs) in \mathbb{R}^2 zo dat

$$\forall D_1 \neq D_2 \in S : \#(D_1 \cap D_2) \in \{0, 1\}$$

↑ raken niet
↑ raken wel



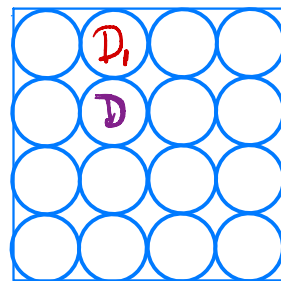
Def Een cirkelstapeling S heet (locaal) compact als

$$\forall D \in S \exists n \geq 3 \text{ en verschillende } D_1, \dots, D_n \in S$$

zo dat $\forall i$:

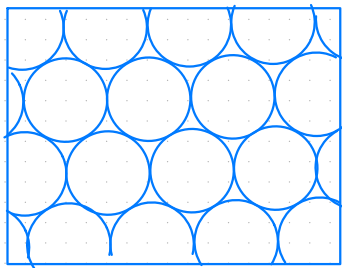
$$D_i \text{ raakt } D \text{ en } D_{(i+1 \bmod n)}.$$

niet compact:



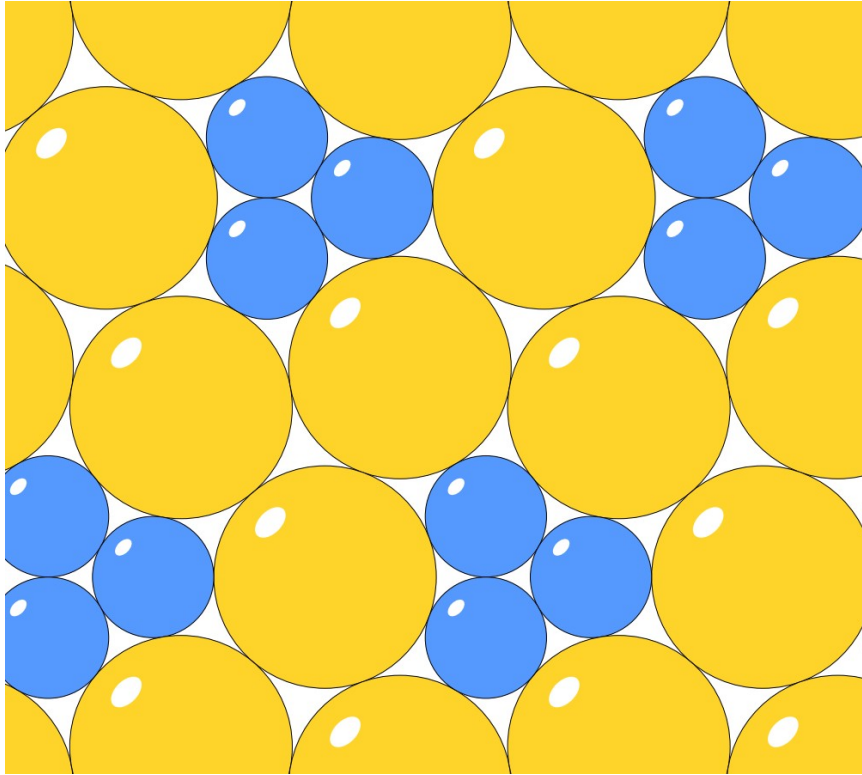
1 Straal

Compacte cirkelstapeling waarin precies één getal als straal voorkomt:

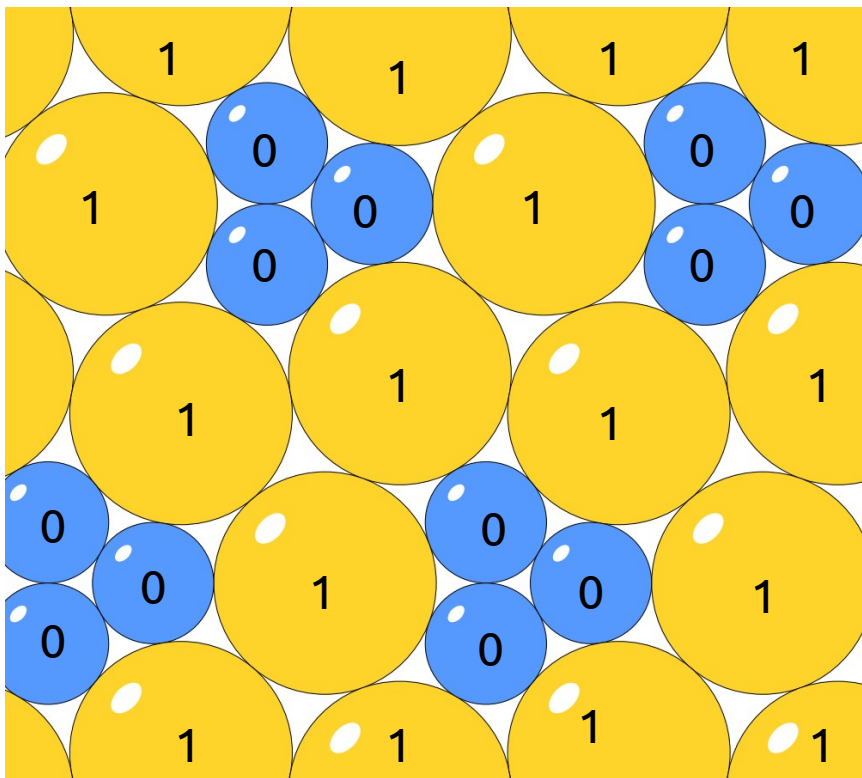


2 stralen

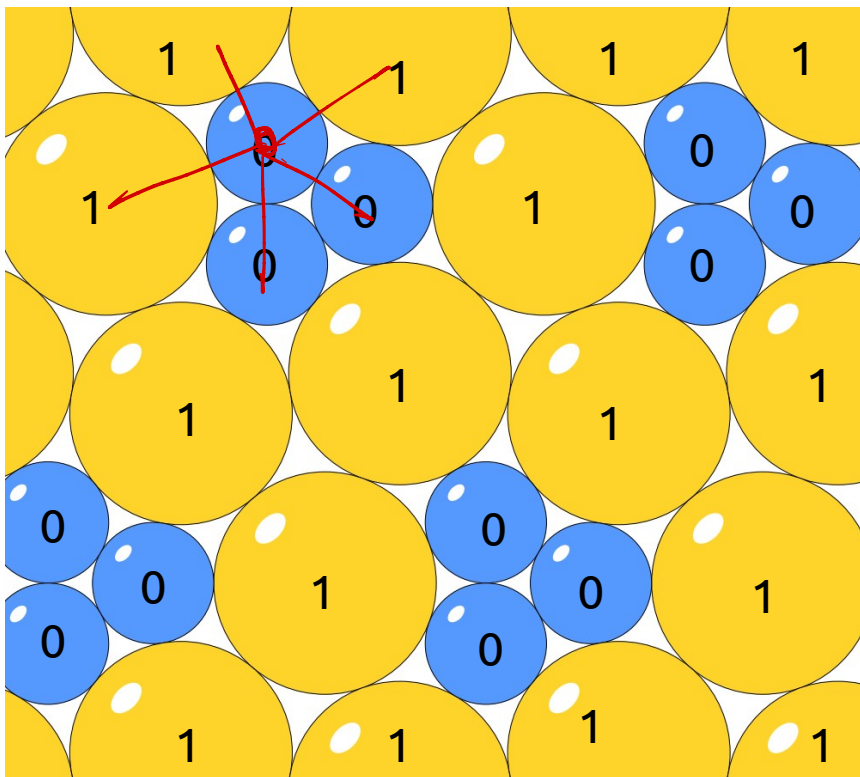
Voor welke $r \in (0, 1)$ is er een compacte cirkelstapeling met stralen r & 1 ?



(Image credit: Thomas Fernique, Amir Hashemi, Olga Sizova)

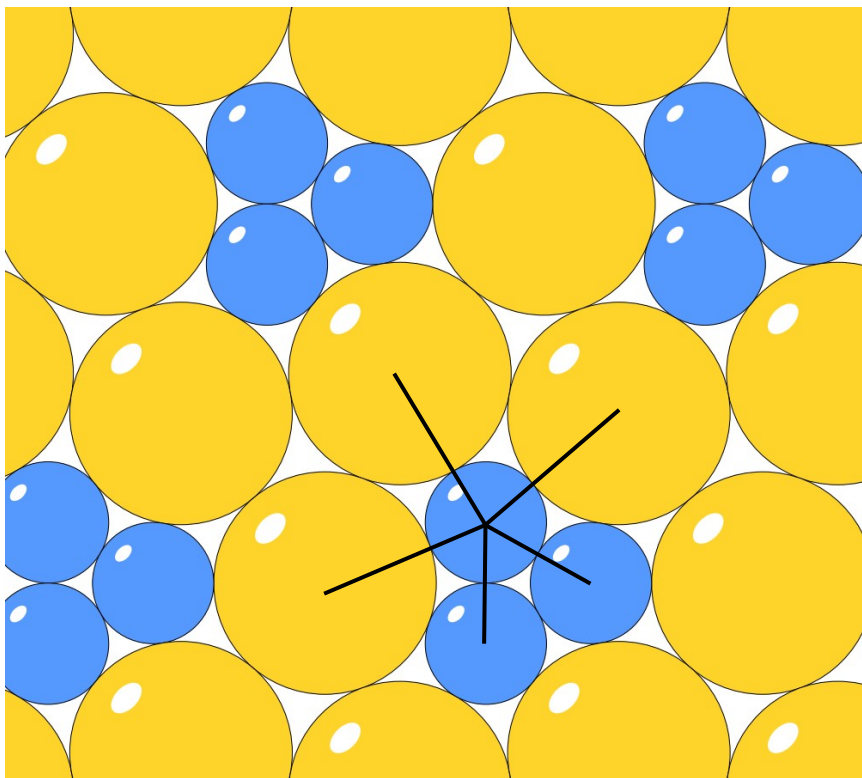


(Image credit: Thomas Fernique, Amir Hashemi, Olga Sizova)

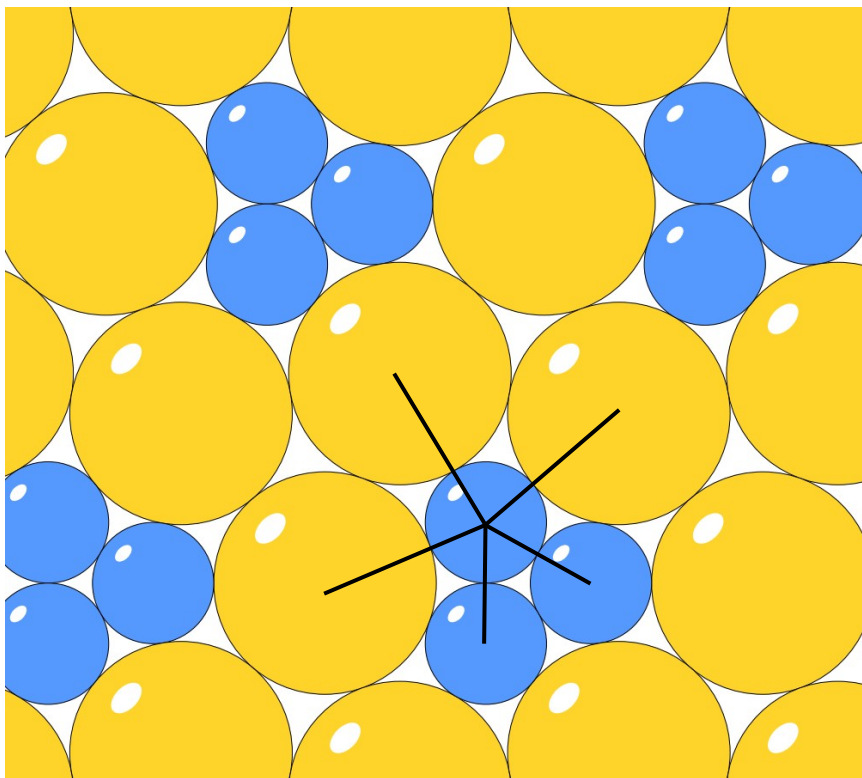


(Image credit: Thomas Fernique, Amir Hashemi, Olga Sizova)

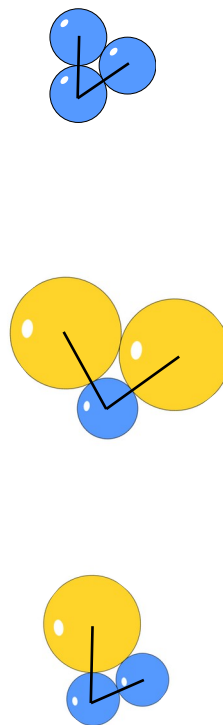
$$\left\{ \begin{array}{l} 0: 11100 \\ 1: 1101100 \end{array} \right\}$$



(Image credit: Thomas Fernique, Amir Hashemi, Olga Sizova)



(Image credit: Thomas Fernique, Amir Hashemi, Olga Sizova)



0:11111
0:11110
0:11100
0:11010
0:11000
0:10100

0:1111
0:1110
0:1100

0:111

Solve for $r := \rho(0) \in (0, 1)$ (exactly)

$$\sum_i \arccos \left(\frac{u_i(r)}{v_i(r)} \right) = 2\pi$$

Solve for $r := \rho(0) \in (0, 1)$ (exactly)

$$\cos \left(\sum_i \arccos \left(\frac{u_i(r)}{v_i(r)} \right) \right) = 1$$

Solve for $r := \rho(0) \in (0, 1)$ (exactly)

$$\sum_i c_i \sqrt{\frac{u_i^*(r)}{v_i^*(r)} \frac{u_i^{**}(r)}{v_i^{**}(r)}} + \sum_j d_j \frac{u_j^{***}(r)}{v_j^{***}(r)} = 0$$

Solve for $r := \rho(0) \in (0, 1)$ (exactly)

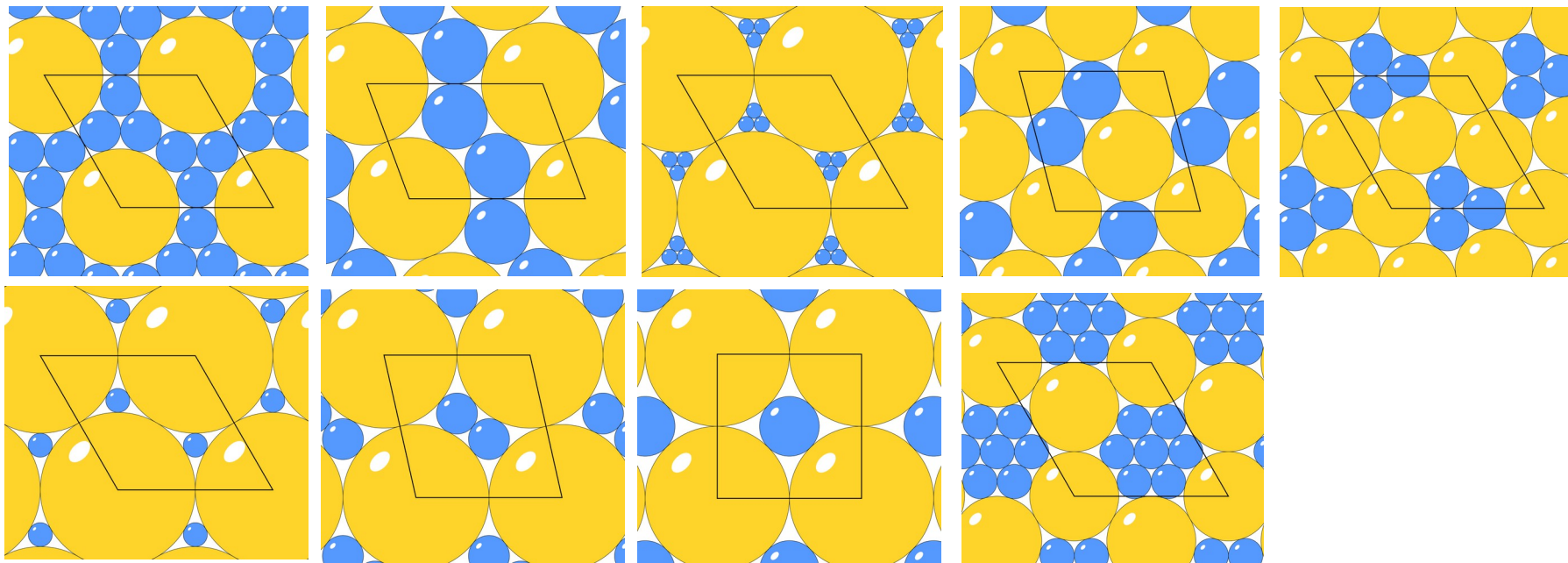
$$\frac{p(r)}{q(r)} = 0$$

Solve for $r := \rho(0) \in (0, 1)$ (exactly)

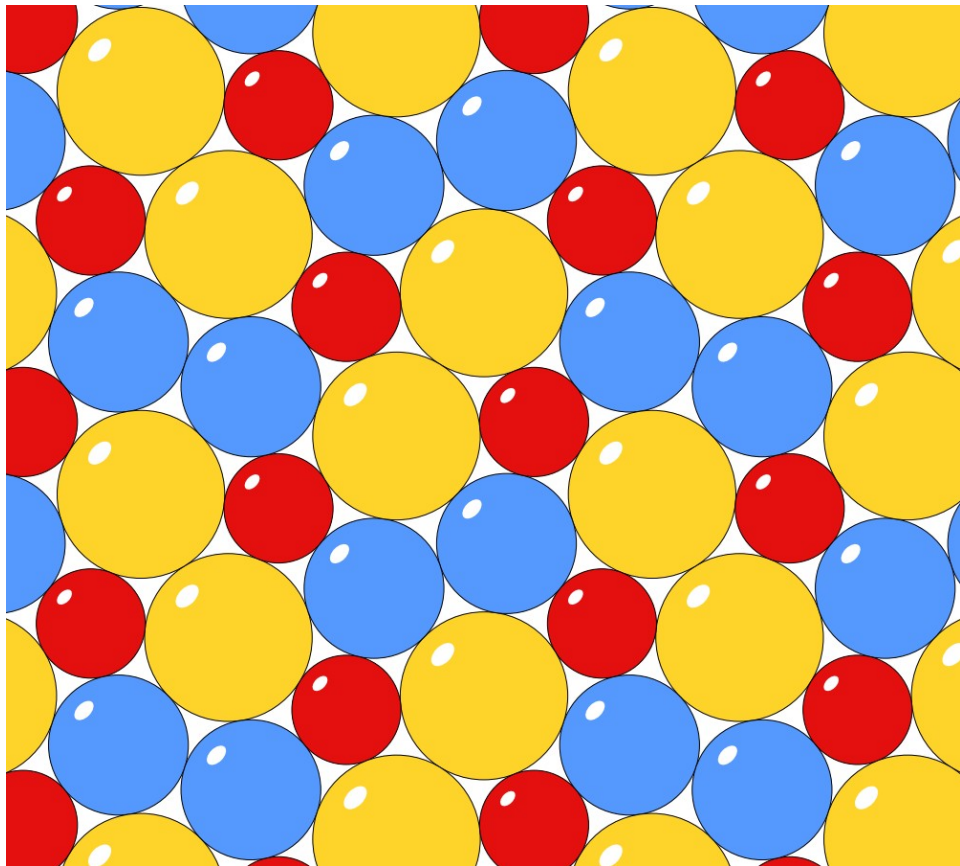
$$p(r) = 0$$

0:11111	$5r^4 + 20r^3 + 10r^2 - 20r + 1$
0:11110	$r^4 - 10r^2 - 8r + 9$
0:11100	$r^8 - 8r^7 - 44r^6 - 232r^5 - 482r^4 - 24r^3 + 388r^2 - 120r + 9$
0:11010	$8r^3 + 3r^2 - 2r - 1$
0:11000	$9r^4 - 12r^3 - 26r^2 - 12r + 9$
0:10100	$r^4 - 28r^3 - 10r^2 + 4r + 1$
0:1111	$r^2 + 2r - 1$
0:1110	$2r^2 + 3r - 1$
0:1100	$r^2 - 10r + 1$
0:111	$3r^2 + 6r - 1$

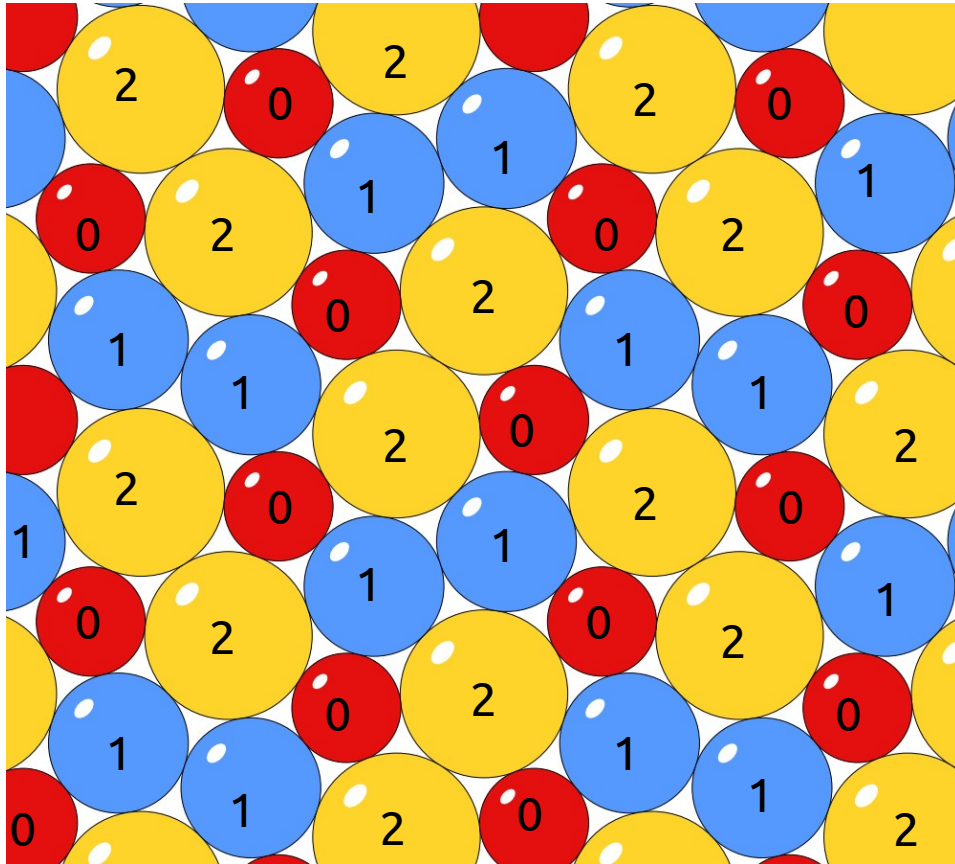
[Kennedy, 2006]



(Image credit: Thomas Fernique, Amir Hashemi, Olga Sizova)

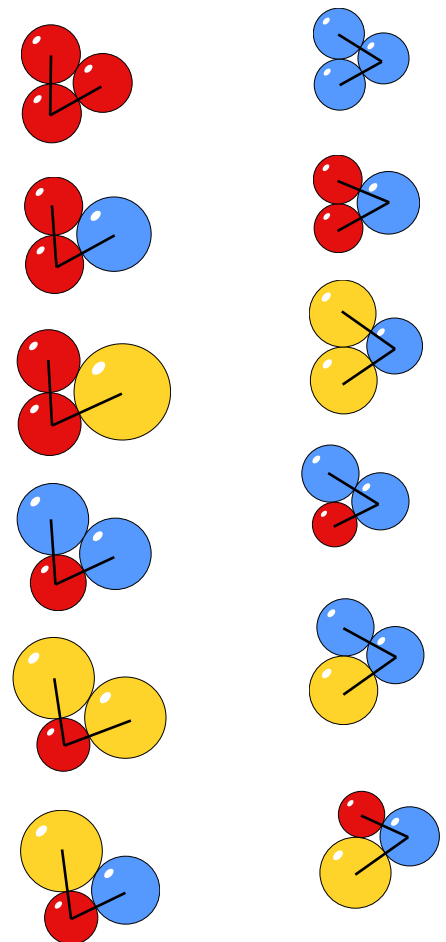
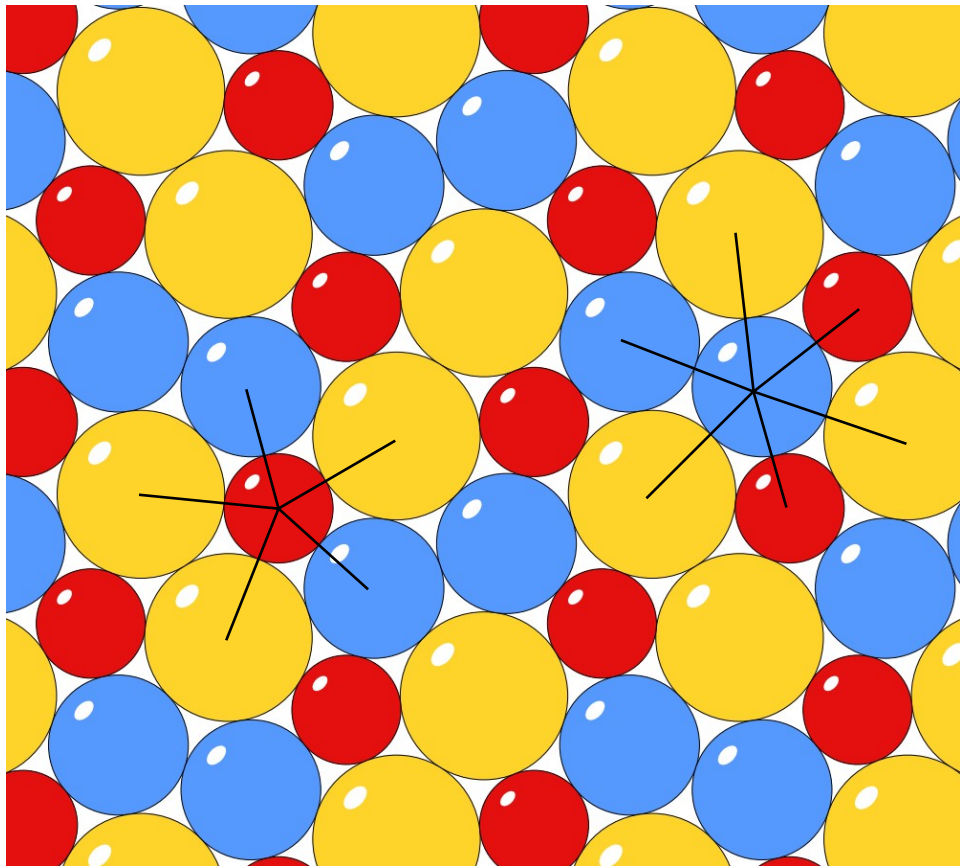


(Image credit: Thomas Fernique, Amir Hashemi, Olga Sizova)



$$\left\{ \begin{array}{l} 0: 22121 \\ 1: 212020 \\ 2: 2011010 \end{array} \right\}$$

(Image credit: Thomas Fernique, Amir Hashemi, Olga Sizova)



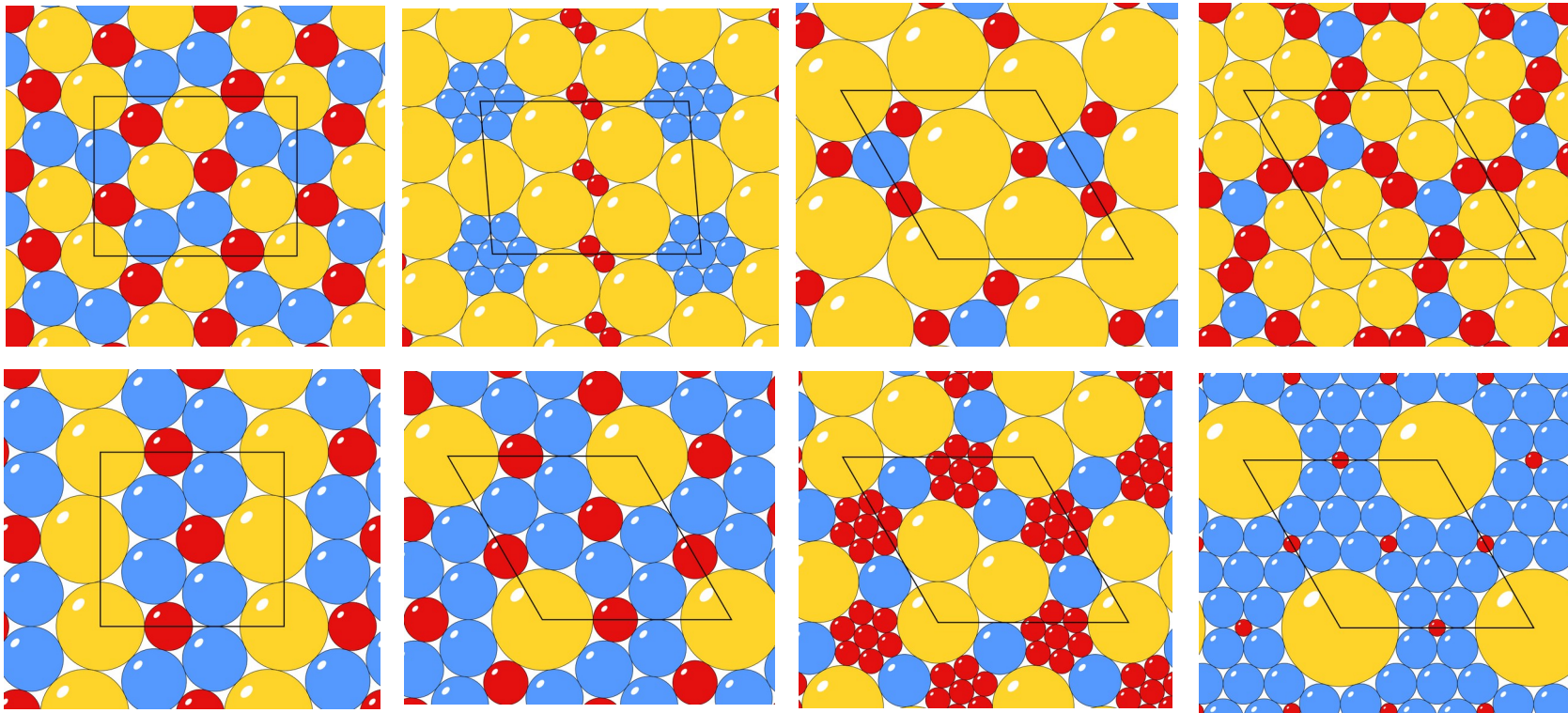
(Image credit: Thomas Fernique, Amir Hashemi, Olga Sizova)

For every pair of codes ($0 : \dots, 1 : \dots$) of the $55 \cdot 1654$ many pairs,
Solve for $0 < s < r < 1$ (exactly) in a system of polynomial
equations

$$p(r, s) = 0$$

$$q(r, s) = 0.$$

Compact packings with three sizes of discs (a sample of the 164)



(Image credit: Thomas Fernique, Amir Hashemi, Olga Sizova) Project: begrijp de bewijzen voor 2 en 3 stralen.

Voorkennis: Algebra 2.

Wat je leert: van alles over het oplossen van polynoomvergelijkingen.

Voorbeelden van mogelijke eigen bijdragen:

* vergelijk de definities van "compact" in de literatuur

* leg het stuk met $\cos(\arccos + \arccos)$ uit in termen van algebra (optelling op cirkelgroep).