Translation surfaces from polygons: topology and geometry

Bachelor project

Supervisor: Olga Lukina, Assistant Professor, Mathematical Institute, Leiden University, email: o.lukina@math.leidenuniv.nl

Pre-requisites: Topology.

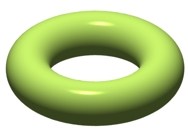

Figure 1: Torus

Take a unit square $[0,1] \times [0,1]$ and identify the opposite edges. The resulting surface is a *torus*, i.e. the surface of a doughnut. The torus is a compact surface of genus one of unit area.

If we subdivide the edges of the square into subintervals (possibly an infinite number of them), and then identify them by translations, we may obtain surfaces with more complicated topology: of genus 2, 3, or even infinite genus. In order to make identifications in a consistent way, we often must remove points (singularities) from the square, which produces *punctures* in the resulting surface. A punctured surfaces is non-compact, and all such surfaces are distinguished by their genus and the number of ends, the latter corresponding to the number of punctures. Singularities are distinguished by how complicated they are, and this leads to the division of translation surfaces into those of *finite type*, and of *infinite type*.

During the project, the student will learn theory of classification for non-compact surfaces by genus and the number of ends, and the basic methods to construct infinite type translation surfaces. The goal of the project is to construct, if possible, new interesting examples of infinite type translation surfaces.

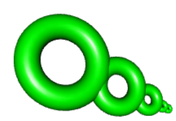

Figure 2: Surface of infinite genus and finite area with one end

References

[1] A. Candel, L. Conlon, *Foliations I*, Chapter 4.2 Ends of manifolds, GSM 23.

[2] V. Delecroix, P. Hubert, F. Valdez, *Infinite translation in the wild*, Chapters 1 and 2, available at https://www.labri.fr/perso/vdelecro/infinite-translation-surfaces-in-the-wild.html.

[3] I. Richards, *On the classification of noncompact surfaces*, Trans. Amer. Math. Soc., 106 (1963), 259–269.