POLYNOMIAL SOLUTION OF POPULATION MODELS

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The basic problem is as follows: you collect some data on the life-cycle of some animal you are interested in (in the original example it was mosquitos, and we counted the numbers of adults and larvae present each day for several days), and then you want to fit a simple population model to this data. This means finding a matrix $M \in \mathrm{Mat}_{a\times b}(\mathbb{R})$ of parameters minimising some error

$$\epsilon\colon \mathrm{Mat}_{a\times b}(\mathbb{R}) \to \mathbb{R}. \tag{1}$$

The amusing observation is that, for a certain class of models, this function ϵ is a polynomial in the entries of the matrix. So instead of using numerical methods to minimise it, you can (at least in principle) find the minima in a completely formal way using Groebner bases.

I wrote a paper with some collaborators from CML a couple of years ago doing this, but we ended using a mix of algebraic and numerical methods (we formally differentiated the polynomial ϵ, then used the derivates as input to a Newton-Raphson minimisation process). The problem we were working with was too big to allow for a completely algebraic solution (at least, I could not get it to work). But if one restricts to a sufficiently-simplified version of the problem it should be possible to get an algebraic solution, which will also have the benefit of finding all minima rather than just a local minimum, etc.

I think this would be an interesting thing to investigate in a bachelor's project. The algebraic content mainly lies in

- a little bit about Hilbert's Nullstellensatz to understand the correspondence between ideals and solution to polynomial equations;
- learning about Groebner bases, and in particular their structure for zero-dimensional ideals.

In principal the entire project could be done theoretically, but I think the results would be a lot more interesting if they can be implemented in a computer algebra package (probably Sage or MAGMA), and used to solve some practical problems. For this co-supervisor from CML may join the project.

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