

Thom's transversality and cobordism theory

Bachelor project, supervisor: F.Pasquotto

Two smooth, closed, k -dimensional manifolds N and N' are called *cobordant* if their disjoint union $N \sqcup N'$ is the boundary ∂M of a smooth, compact, $(k + 1)$ -dimensional manifold M .

In the 1950's, Pontryagin showed that a particular flavour of cobordism, namely *framed cobordism*, is an equivalence relation on the set of smooth, k -dimensional submanifolds of \mathbb{R}^{n+k} with smoothly trivialized normal bundle, and that the corresponding equivalence classes admit a group structure. Moreover, he proved that these groups are isomorphic to certain homotopy groups of spheres.

Subsequently, Thom greatly developed the theory of cobordism: in particular, he introduced the notion of *transversality*, as a generalisation of the concept of a regular value for smooth maps between manifolds. He also defined the *Thom space* of a vector bundle, and linked more general cobordism groups to homotopy groups of Thom spaces of so called *universal (or classifying) vector bundles*.

In this project, the student will first get acquainted with the basics concepts of *differential topology* (regular values and Sard's theorem, mapping degree...), then study different notions of cobordism and the *Pontryagin-Thom construction*, which relates cobordism groups of smooth manifolds to homotopy groups. Depending on time and ambition, the project could be extended to take a peek into the actual computations of the cobordism groups.

Prerequisites: topology and differentiable manifolds. As the project progresses, some notions of algebraic topology (higher homotopy groups) will become useful, but can also be acquired as an integral part of the project.

Literature:

- J.W. Milnor, *Topology from the differentiable viewpoint*, Princeton University Press (Rev. ed., 1997)
- J.W. Milnor and J.D. Stasheff, *Characteristic classes*, Princeton University Press (1974)