Dehn twists and mapping class groups of surfaces

Bachelor project, supervisor: F.Pasquotto

Given a surface S, homeomorphisms from S to itself naturally form a group under composition. This group, however, is too large to be studied directly. Therefore one usually tries to understand its quotient by a suitable equivalence relation, namely *isotopy*. An isotopy between two homeomorphisms of S is a homotopy $F : S \times [0,1] \to S$ such that $F(\cdot, t) : S \to S$ is a homeomorphism for all $t \in [0,1]$. The *mapping class group of S* is defined to be the quotient of the group of homeomorphisms of S by isotopy, and this quotient is denoted by $\mathrm{Mod}(S)$. Mapping class groups are important algebraic invariants of topological spaces, and their structure provides crucial information about, among others, invariants of 3-dimensional manifolds and properties of moduli space of surfaces.

Dehn twists are particularly simple elements of the mapping class group and their properties can be understood by studying their action on simple closed curves. In fact, it turns out that a finite number of Dehn twists generate the mapping class group. This result is due to Dehn (1938) and was later improved by Lickorish (1964).

In the first part of this project, the student will study the definition of mapping class group (for closed surfaces, but also in the case of surfaces with boundary) and compute some simple examples. In the second part, they will concentrate on the defintion of Dehn twists and the proof of the Dehn-Lickorish theorem.

Depending on time and the student's ambition, the project can be extended to include the relationship between mapping class groups of closed surfaces and the braid groups.

Prerequisites: algebra and topology. As the project progresses, some notions of algebraic topology and differential topology will become useful, but can also be acquired as an integral part of the project.

Literature:

- J. Stillwell, *Geometry of surfaces*, Springer (1992)
- B. Farb and D. Margalit, *A Primer on Mapping Class Groups*, Princeton University Press (2012)