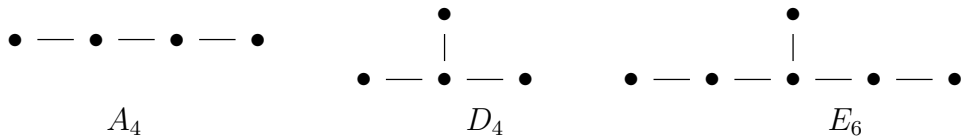
# BACHELOR PROJECT: QUIVER REPRESENTAION

PROPOSED BY: YAJNASENI DUTTA




These graphs, more generally the ADE graphs appear in an array of mathematical areas, the study of isolated singularities, Lie algebras and representation theory. In this project we will focus on the representation theoretic side of the story.

**Prerequisite.** Basic course on representation theory; it is also fine, if taken in parallel. Having seen some projective modules, a bit of homological algebra will go a long way. Affine algebraic geometry will be very helpful, for example the algebraic curves class.

**Overview.** What is representation theory? It is the study of various objects (e.g., groups, algebras, even graphs!) by representing its elements as linear transformation of vector spaces.

Given a quiver (i.e., a finite directed graph), we can represent its vertices by finite dimensional vector spaces and arrows by linear transformations between the corresponding vector spaces. A morphism between two quiver representations is given by a collection of linear transformations, one for each vertex such that they are compatible with the linear transformations in the representations. The *dimension vector* associated to a quiver representation is the tuple that is obtained by considering the dimensions of the vector spaces at the vertices.

The goal of representation theory is to classify representations up-to isomorphisms. To this end Krull–Schmidt theorem tells us that any finite dimensional quiver representation is isomorphic to a direct sum of indecomposable representations. Therefore, we only need to classify indecomposable ones.

**Goal.** The main goal of this project would be to understand Gabriel's theorem (1972): A quiver has only finitely many isomorphism classes of indecomposable representations of any prescribed dimension vector if and only if each connected component of its underlying undirected graph is an ADE graph.

**References.** An excellent overview of the material is in https://www-fourier.ujf-grenoble.fr/~mbrion/notes_quivers_rev.pdf. Aside from this, https://www.math.uni-bielefeld.de/~wcrawley/quivlecs.pdf and the book [Quiver Representation, R. Schiffler] are excellent sources.

**Where else could we go?** Albeit infinite, one could still consider examples of non-ADE quivers and try to classify them. One could try some examples that are not in any of the notes, for instance, what happens if we introduce arrows in the opposite direction (classification of these leads to interesting Nakajima quiver varieties)? Is there a systematic way to do this? There are applications of quiver representations to neural networks https://arxiv.org/pdf/2109.14589. If there are enough interest, we could explore this direction together.