SHAPE THEORY

Shape theory [2] can be described as the extension of algebraic topology from the category $\mathcal{CW}$ of finite CW-complexes to the category $\mathcal{M}$ of all compact metrisable spaces. Every compact metrisable space is an inverse limit of finite CW-complexes. Easily visualisable examples are the Hawaiian earring [1] and the Warsaw circle. In this project we will investigate extension properties of familiar functors from algebraic topology, such as homotopy groups, (co)homology, and K-theory.

If a functor is continuous with respect to inverse limits of CW-complexes, then its definition extends, by continuity, to all compact metrisable spaces. Topological K-theory is an example of such a functor. In the absence of such continuity, a given functor may admit multiple extensions. This is the case for the fundamental group. For (co)homology, the Eilenberg-Steenrod axioms provide a characterization on $\mathcal{CW}$, but this characterisation breaks down on $\mathcal{M}$.

Models for functors that are equivalent on $\mathcal{CW}$ may no longer be equivalent on $\mathcal{M}$. For instance, on $\mathcal{CW}$ the fundamental group can be described either via equivalence classes of pointed loops, or as the group of deck transformations of the universal cover.

The goal of this project is to gain a clear understanding of the continuity properties of various functors. Furthermore, we aim to describe their (possibly) distinct extensions to $\mathcal{M}$ in a number of concrete examples. If time allows, we will look at natural transformations between them (such as the Hurewicz homomorphisms and Chern characters).

References

[1] B.de Smit, *The fundamental group of the Hawaiian earring is not free*, International Journal of Algebra and Computation Vol. 2, No. 1 (1992), 33–37.

[2] P.S. Gevorgyan, *Shape theory*, Journal of Mathematical Sciences, Vol. 259, (2021), 583–627.