Algebraic Curves

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In the bachelor/master course 'Algebraic Curves' (in the Autumn term) we briefly mentioned many topics worthy of much more detailed study. In this project we will look into one of these areas in a little more detail. Below I list a number of (non-disjoint) possibilities, but there are many others, and I'm very happy to talk to students interested in taking any part of the course further. For students who have not attended 'Algebraic Curves' it may be possible to learn the necessary background as part of the project.

1 Resolution of singularities for curves.

We saw the basic definition of a blowup, and stated that they give an algorithm for resolution of singularities for curves, but we did not prove this. There are a number of proofs in the literature (see e.g the book of Kollar on resolution of singularities); the goal of this project would be to understand (at least) one of them in detail.

2 Resolution for surfaces

The aim will be to understand blowups of surfaces, and to see why simply blowing up the singular locus does not yield a resolution algorithm.

3 Dimensions of spaces of curves

We wrote down explicit formulae for the dimensions of the vector spaces of plane curves of given degree and allowing certain types of singularities at certain points. How does this story change when we look at curves in $\mathbb{P}^3$, or curves in an interesting projective surface?

4 Counting curves

Building on the previous proposal, by restricting another invariant (the *genus*) one can end up with sets of curves that are not vector spaces; by choosing the invariants carefully one can get finite sets of curves, whose cardinalities are interesting invariants with connections to mathematical physics, classical geometry etc.