# Bachelor projects

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## 1. Invariants of Legendrian and transverse knots in standard contact space

Contact structures on manifolds and Legendrian and transverse knots in them are very natural geometric objects, with connections to different areas of mathematics and physics, such as (low-dimensional) topology and dynamics. They already appear in work of Huygens on optics and work of Lie on partial differential equations.

This project concentrates on Legendrian and transverse knots in dimension three and their invariants and classification. We plan to define and compute the Thurston-Bennequin and the rotation number for Legendrian knots, and learn to compute them using suitable projections. We will learn that these invariant classify Legendrian unknots, but some time fail to distinguish more complicated knots.

Towards the end of the project there will hopefully be time to discuss some concrete applications of this theory, such as Geiges' contact geometric proof of the Whitney-Graustein theorem (about homotopy classes of closed curves in the plane).

**Literature:**


- H. Geiges, *An introduction to contact topology*, Cambridge University Press (2008)
- H. Geiges, *A contact geometric proof of the Witney-Graustein theorem*, Enseign. Math. 55 (2009)
- D. Fuchs and S. Tabachnikov, *Invariants of Legendrian and transverse knots in the standard contact space*, Topology 36 (1997)


## 2. Systolic inequalities on surfaces

Given a Riemannian metric $g$ on a non simply connected surface $\Sigma$, there exists a non-contractible loop of minimal length called a *systole* of $g$. The length of this loop is denoted by $\mathrm{sys}(g)$ and it gives us one way to measure the "size" of a surface. A natural question to ask is the following: what is the relationship between sys($g$) and the total area of $\Sigma$, namely $\mathrm{Area}_g(\Sigma)$? This leads us to the notion of *systolic ratio* of a surface.

We will start the project by learning about geometry of surfaces (metric, curvature, geodesics). The ultimate goal of the project is to discuss existence of upper bounds for the systolic ratio on several surfaces, and in the case of the torus construct the metric which maximizes this ratio.

**Literature:**


- J. Stillwell, *Geometry of surfaces*, Springer (1992)
- M. Berger, *What is... a systole?*, Notices of the AMS 55 (2008)
- G. Benedetti, *First steps into the world of systolic inequalities*, notes written for the DDT&G seminar (2020)